The Greeks: Derivatives of Option Prices

MATH 472 *Financial Mathematics*

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Sensitivity Analysis

- In calculus a **derivative** gives you a measure of the rate of change of a dependent variable as an independent variable is changed.
- In the world a finance an option is an example of a **derivative**, any financial instrument whose value is *derived* from that of an underlying security.
- Here we will calculate the partial derivatives (in the sense of calculus) of option value formulas. These partial derivatives will allow us to determine how sensitive the values of options are to changes in independent variables and parameters.
- In finance these partial derivatives are referred to as “the Greeks”.
- Unless otherwise specified, all results discussed are valid only for non-dividend paying securities.

Black-Scholes Option Pricing Formulas

$$
\begin{aligned}
w &= \frac{\ln(S/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}} \\
C^e(S, t) &= S\,\Phi\,(w) - K\,e^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T - t}\right) \\
P^e(S, t) &= K\,e^{-r(T-t)}\Phi\left(\sigma\sqrt{T - t} - w\right) - S\,\Phi\,(-w)
\end{aligned}
$$

Cumulative Distribution Function

The function $\Phi(w)$ is the cumulative distribution function

$$\Phi(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{w} e^{-x^2/2}\, dx$$

which by the Fundamental Theorem of Calculus has derivative

$$\Phi'(w) = \phi(w) = \frac{1}{\sqrt{2\pi}} e^{-w^2/2}.$$

Partial Derivatives of w

$$
\begin{aligned}
w &= \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \\
\frac{\partial w}{\partial t} &= \frac{1}{2\sigma\sqrt{T-t}}\left(\frac{\ln(S/K)}{T-t} - r - \frac{\sigma^2}{2}\right) \\
\frac{\partial w}{\partial S} &= \frac{1}{\sigma S\sqrt{T-t}} \\
\frac{\partial w}{\partial r} &= \frac{\sqrt{T-t}}{\sigma} \\
\frac{\partial w}{\partial \sigma} &= \sqrt{T-t} - \frac{w}{\sigma}
\end{aligned}
$$

Important Identity

Claim:

$$S\,\phi(w) = Ke^{-r(T-t)}\,\phi\left(w - \sigma\sqrt{T-t}\right)$$

Theta Θ (1 of 3)

Theta Θ is the partial derivative with respect to time t.

Time is the only independent variable we are certain will change before expiry. It is also the only deterministic independent variable.

Theta Θ (2 of 3)

$$
\begin{aligned}
C^e &= S\Phi(w) - Ke^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) \\
\frac{\partial C^e}{\partial t} &= S\phi(w)\frac{\partial w}{\partial t} - rKe^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) \\
&\quad - Ke^{-r(T-t)}\phi\left(w - \sigma\sqrt{T-t}\right)\left[\frac{\partial w}{\partial t} + \frac{\sigma}{2\sqrt{T-t}}\right]
\end{aligned}
$$

Theta Θ (2 of 3)

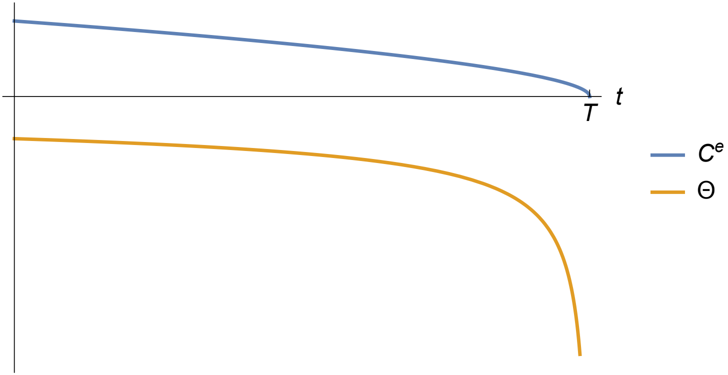
$$\begin{aligned}
C^e &= S\Phi(w) - Ke^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) \\
\frac{\partial C^e}{\partial t} &= S\phi(w)\frac{\partial w}{\partial t} - rKe^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) \\
&\quad - Ke^{-r(T-t)}\phi\left(w - \sigma\sqrt{T-t}\right)\left[\frac{\partial w}{\partial t} + \frac{\sigma}{2\sqrt{T-t}}\right] \\
&= S\phi(w)\frac{\partial w}{\partial t} - rKe^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) \\
&\quad - S\phi(w)\left[\frac{\partial w}{\partial t} + \frac{\sigma}{2\sqrt{T-t}}\right]
\end{aligned}$$

Theta Θ (2 of 3)

$$\begin{aligned}
C^e &= S\Phi(w) - Ke^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) \\
\frac{\partial C^e}{\partial t} &= S\phi(w)\frac{\partial w}{\partial t} - rKe^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) \\
&\quad - Ke^{-r(T-t)}\phi\left(w - \sigma\sqrt{T-t}\right)\left[\frac{\partial w}{\partial t} + \frac{\sigma}{2\sqrt{T-t}}\right] \\
&= S\phi(w)\frac{\partial w}{\partial t} - rKe^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) \\
&\quad - S\phi(w)\left[\frac{\partial w}{\partial t} + \frac{\sigma}{2\sqrt{T-t}}\right] \\
&= -\frac{\sigma S}{2\sqrt{T-t}}\phi(w) - r\,K\,e^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) < 0
\end{aligned}$$

Illustration

The value of a European Call decreases as expiry approaches (all other variables and parameters being constant).



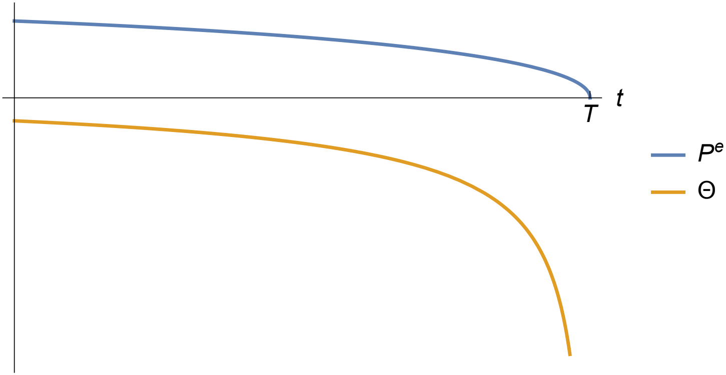
Theta Θ (3 of 3)

For a European Put:

$$
\begin{aligned}
\frac{\partial P^e}{\partial t} &= r\,K\,e^{-r(T-t)}\Phi\left(\sigma\sqrt{T-t}-w\right) + S\,\phi\,(-w)\frac{\partial w}{\partial t} \\
&\quad - K\,e^{-r(T-t)}\phi\left(\sigma\sqrt{T-t}-w\right)\left[\frac{\sigma}{2\sqrt{T-t}} + \frac{\partial w}{\partial t}\right] \\
&= r\,K\,e^{-r(T-t)}\Phi\left(\sigma\sqrt{T-t}-w\right) - \frac{\sigma S}{2\sqrt{T-t}}\phi\,(w)
\end{aligned}
$$

Illustration

The value of a European Put decreases as expiry approaches (all other variables and parameters being constant).



Delta Δ (1 of 2)

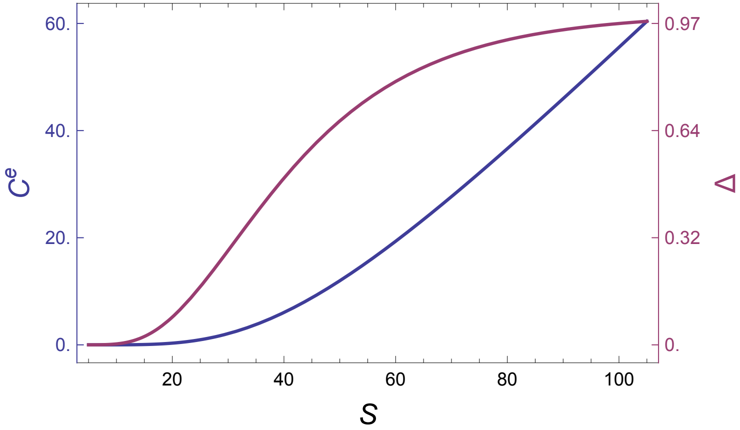
Δ was involved in the derivation of the Black-Scholes PDE and is defined to be the partial derivative with respect to the price of the security.

$$
\begin{aligned}
C^e &= S\,\Phi(w) - K\,e^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) \\
\frac{\partial C^e}{\partial S} &= \Phi(w) + S\,\phi(w)\frac{\partial w}{\partial S} - K\,e^{-r(T-t)}\phi\left(w - \sigma\sqrt{T-t}\right)\frac{\partial w}{\partial S} \\
&= \Phi(w) > 0
\end{aligned}
$$

Question: what is the range of Delta for a European call option?

Illustration

Consider a European Call with $K = 50$, $T = 1$, $r = 0.10$, and $\sigma = 0.50$.



Delta Δ (2 of 2)

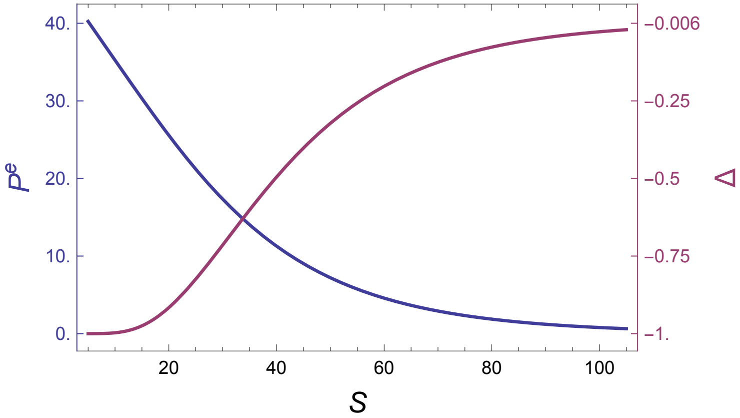
Recall the Put-Call Parity formula:

$$\begin{aligned}
P^e + S &= C^e + K\, e^{-r(T-t)} \\
\frac{\partial}{\partial S}[P^e + S] &= \frac{\partial}{\partial S}[C^e + K\, e^{-r(T-t)}] \\
\frac{\partial P^e}{\partial S} + 1 &= \frac{\partial C^e}{\partial S} \\
\frac{\partial P^e}{\partial S} &= \Phi\,(w) - 1 < 0
\end{aligned}$$

Question: what is the range of Delta for a European put option?

Illustration

Consider a European Put with $K = 50$, $T = 1$, $r = 0.10$, and $\sigma = 0.50$.



Example (1 of 2)

The current price of a non-dividend-paying stock is \$77 and its volatility is 35% per year. The risk-free interest rate is 3.25% per year. A portfolio is constructed consisting of one six-month European call option with a strike price of \$80 and the cash obtained from shorting Δ shares of the stock. The portfolio's value is non-random. What is Δ?

Example (2 of 2)

The assumption the portfolio's value is non-random is the assumption

$$(\Delta)S - C^e = \left(\frac{\partial C^e}{\partial S}\right) S - C^e = 0$$

made in deriving the Black-Scholes equation.

$$\begin{aligned} S &= 77 \quad \sigma = 0.35 \quad T = \frac{6}{12} \\ r &= 0.0325 \quad K = 80 \quad t = 0 \end{aligned}$$

Using these values

$$\begin{aligned} w &= \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \\ \frac{\partial C^e}{\partial S} &= \Delta = \Phi(w) \end{aligned}$$

Example (2 of 2)

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$$(\Delta)S - C^e = \left(\frac{\partial C^e}{\partial S}\right) S - C^e = 0$$

made in deriving the Black-Scholes equation.

$$\begin{aligned} S &= 77 \quad \sigma = 0.35 \quad T = \frac{6}{12} \\ r &= 0.0325 \quad K = 80 \quad t = 0 \end{aligned}$$

Using these values

$$\begin{aligned} w &= \frac{\ln(S/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}} \approx 0.0349666 \\ \frac{\partial C^e}{\partial S} &= \Delta = \Phi(w) \approx 0.513947 \end{aligned}$$

Example (1 of 2)

Suppose a portfolio consists of a share of stock worth \$75 and a European Put option on that stock with a strike price of \$73 and expiry in 3 months. Assume the risk-free interest rate is 10% and the volatility of the stock price is 30%.

Find the Delta of the portfolio consisting of the stock and the option.

Example (2 of 2)

The Delta of the portfolio is

$$\frac{\partial}{\partial S}\left[S + P^e\right] = 1 + \Phi\left(w\right) - 1 = \Phi\left(w\right)$$

calculated using the variables and parameters below.

$$\begin{array}{rclccc} S & = & 75 & \sigma = 0.30 & T = 3/12 \\ r & = & 0.10 & K = 73 & t = 0 \end{array}$$

$$w = \frac{\ln(S/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}$$

Example (2 of 2)

The Delta of the portfolio is

$$\frac{\partial}{\partial S}\left[S + P^e\right] = 1 + \Phi\left(w\right) - 1 = \Phi\left(w\right)$$

calculated using the variables and parameters below.

$$\begin{array}{lll} S = 75 & \sigma = 0.30 & T = 3/12 \\ r = 0.10 & K = 73 & t = 0 \end{array}$$

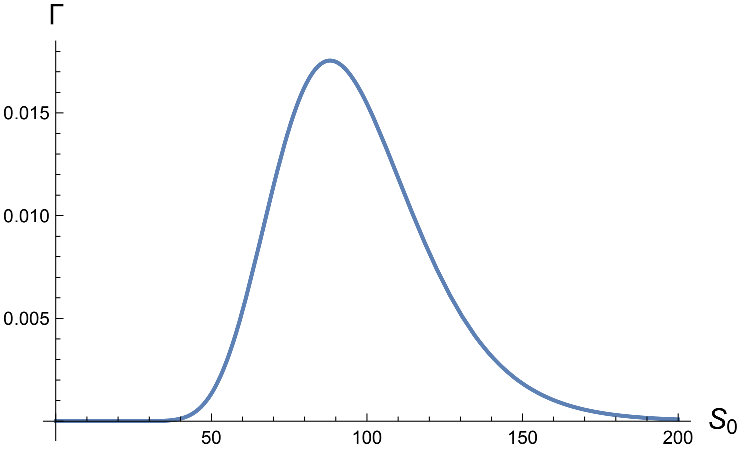
$$\begin{aligned} w &= \frac{\ln(S/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}} \approx 0.421858 \\ \Phi\left(w\right) &\approx 0.663436 \end{aligned}$$

Gamma Γ

Gamma is the second partial derivative with respect to S, thus

$$\begin{aligned}
\Gamma &= \frac{\partial}{\partial S}\left[\Phi\left(w\right)\right] \\
&= \phi\left(w\right)\frac{\partial w}{\partial S} \\
\frac{\partial^2 C^e}{\partial S^2} = \frac{\partial^2 P^e}{\partial S^2} &= \frac{e^{-w^2/2}}{\sigma S\sqrt{2\pi(T-t)}} > 0.
\end{aligned}$$

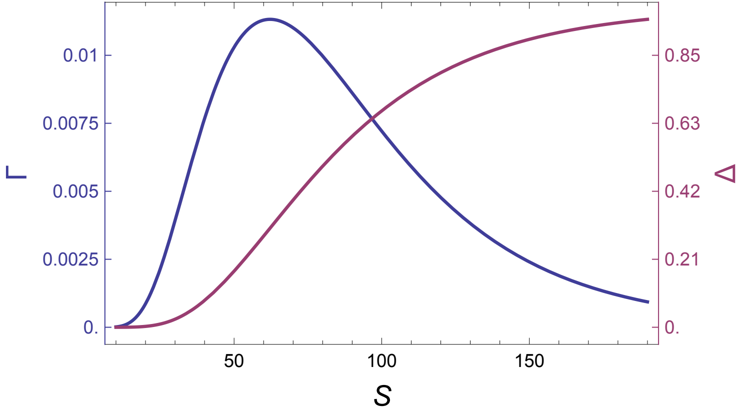
Gamma *vs.* S_0



$$K = 100, \quad \sigma = 0.25 \quad T = 1 \quad r = 0.0325$$

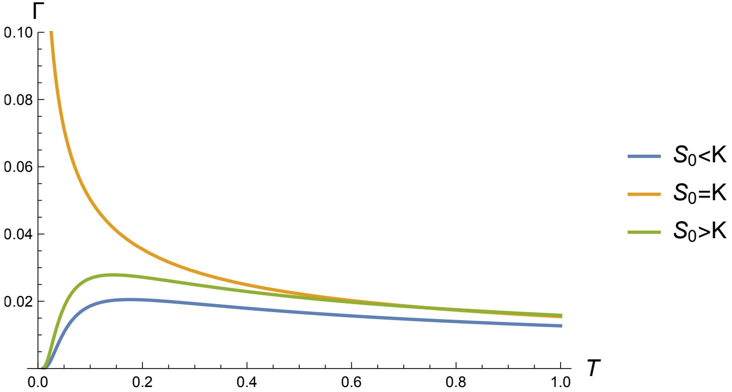
Gamma and Delta

For options far in-the-money or out-of-the-money, there is little change in Δ and thus Γ is nearly zero.



Gamma and At-the-Money Options

Consider an at-the-money option ($S = K$), how does Gamma behave as expiry approaches?

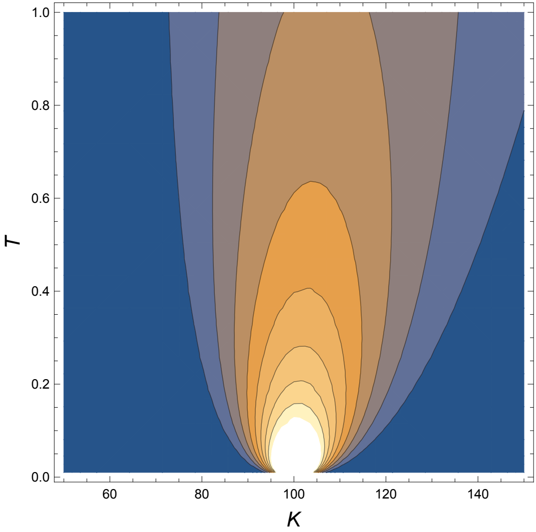


Solution

When $S = K$,

$$\begin{aligned}\lim_{t \to T^-} \Gamma &= \lim_{t \to T^-} \frac{e^{-((r+\sigma^2/2)(T-t))^2/(2\sigma^2(T-t))}}{\sigma K \sqrt{2\pi(T-t)}} \\ &= \lim_{t \to T^-} \frac{e^{-(r+\sigma^2/2)^2(T-t)/(2\sigma^2)}}{\sigma K \sqrt{2\pi(T-t)}} \\ &= \infty.\end{aligned}$$

Gamma *vs.* K and T



$$S_0 = 100, \quad \sigma = 0.25 \quad r = 0.0325$$

Relationships Between Δ, Θ, and Γ

Remember the Black-Scholes PDE:

$$r\,F = F_t + r\,S\,F_S + \frac{1}{2}\sigma^2 S^2 F_{SS}$$

Since $F_t = \Theta$, $\Delta = F_S$, and $F_{SS} = \Gamma$ then the Black-Scholes equation can be thought of as

$$r\,F = \Theta + r\,S\,\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma.$$

Changes in Option Values

Using differentials we can approximate changes in the prices of options as underlying variables and parameters change.

Let $F(S,t)$ be the value of an option at time t when the value of the underlying security is S, then we have the following approximations.

$$\begin{aligned}
F(S, t+\delta t) &\approx F(S,t) + (\Theta)\delta t \\
F(S+\delta S, t) &\approx F(S,t) + (\Delta)\delta S \\
F(S+\delta S, t) &\approx F(S,t) + (\Delta)\delta S + \frac{1}{2}(\Gamma)(\delta S)^2 \\
F(S+\delta S, t+\delta t) &\approx F(S,t) + (\Theta)\delta t + (\Delta)\delta S + \frac{1}{2}(\Gamma)(\delta S)^2
\end{aligned}$$

Example (1 of 3)

A six-month call option with a strike price of \$100 on a stock currently valued at \$99 and having a volatility of $\sigma = 0.40$ costs \$12.4911. The risk-free interest rate is $r = 0.08$. Estimate the value of the option at five months to expiry.

Example (1 of 3)

A six-month call option with a strike price of \$100 on a stock currently valued at \$99 and having a volatility of $\sigma = 0.40$ costs \$12.4911. The risk-free interest rate is $r = 0.08$. Estimate the value of the option at five months to expiry.

$$\begin{aligned}
C^e(S, t + \delta t) &\approx C^e(S, t) + (\Theta)\delta t \\
C^e(S, \delta t) &\approx C^e(S, 0) + (\Theta)\delta t \\
C^e(99, 1/12) &\approx C^e(99, 0) + (\Theta)(1/12) \\
&= 12.4911 + \frac{-14.5686}{12} \\
&= 11.2771
\end{aligned}$$

Example (1 of 3)

A six-month call option with a strike price of \$100 on a stock currently valued at \$99 and having a volatility of $\sigma = 0.40$ costs \$12.4911. The risk-free interest rate is $r = 0.08$. Estimate the value of the option at five months to expiry.

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C^e(99, 1/12) &\approx C^e(99, 0) + (\Theta)(1/12) \\
&= 12.4911 + \frac{-14.5686}{12} \\
&= 11.2771
\end{aligned}$$

For comparison, the exact value is $C^e(99, 1/12) = \$11.2322$.

Example (2 of 3)

A six-month call option with a strike price of \$100 on a stock currently valued at \$99 and having a volatility of $\sigma = 0.40$ costs \$12.4911. The risk-free interest rate is $r = 0.08$. Estimate the value of the option using Delta if the value of the stock increases to \$101.

Example (2 of 3)

A six-month call option with a strike price of \$100 on a stock currently valued at \$99 and having a volatility of $\sigma = 0.40$ costs \$12.4911. The risk-free interest rate is $r = 0.08$. Estimate the value of the option using Delta if the value of the stock increases to \$101.

$$\begin{aligned} C^e(S+\delta S, t) &\approx C^e(S, t) + (\Delta)\delta S \\ C^e(101, 0) &\approx C^e(99, 0) + (\Delta)(2) \\ &= 12.4911 + (0.597666)(2) \\ &= 13.6865 \end{aligned}$$

Example (2 of 3)

A six-month call option with a strike price of \$100 on a stock currently valued at \$99 and having a volatility of $\sigma = 0.40$ costs \$12.4911. The risk-free interest rate is $r = 0.08$. Estimate the value of the option using Delta if the value of the stock increases to \$101.

$$\begin{aligned} C^e(S + \delta S, t) &\approx C^e(S, t) + (\Delta)\delta S \\ C^e(101, 0) &\approx C^e(99, 0) + (\Delta)(2) \\ &= 12.4911 + (0.597666)(2) \\ &= 13.6865 \end{aligned}$$

For comparison, the exact value is $C^e(101, 0) = \$13.7137$.

Example (3 of 3)

A six-month call option with a strike price of \$100 on a stock currently valued at \$99 and having a volatility of $\sigma = 0.40$ costs \$12.4911. The risk-free interest rate is $r = 0.08$. Estimate the value of the option using Delta and Gamma if the value of the stock increases to \$101.

Example (3 of 3)

A six-month call option with a strike price of \$100 on a stock currently valued at \$99 and having a volatility of $\sigma = 0.40$ costs \$12.4911. The risk-free interest rate is $r = 0.08$. Estimate the value of the option using Delta and Gamma if the value of the stock increases to \$101.

$$\begin{aligned}
C^e(S+\delta S, t) &\approx C^e(S,t) + (\Delta)\delta S + \frac{1}{2}(\Gamma)(\delta S)^2 \\
C^e(101, 0) &\approx C^e(99, 0) + (\Delta)(2) + \frac{1}{2}(\Gamma)(4) \\
&= 12.4911 + (0.597666)(2) + \frac{1}{2}(0.0138181)(4) \\
&= 13.7141
\end{aligned}$$

Example (3 of 3)

A six-month call option with a strike price of \$100 on a stock currently valued at \$99 and having a volatility of $\sigma = 0.40$ costs \$12.4911. The risk-free interest rate is $r = 0.08$. Estimate the value of the option using Delta and Gamma if the value of the stock increases to \$101.

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C^e(101, 0) &\approx C^e(99, 0) + (\Delta)(2) + \frac{1}{2}(\Gamma)(4) \\
&= 12.4911 + (0.597666)(2) + \frac{1}{2}(0.0138181)(4) \\
&= 13.7141
\end{aligned}$$

For comparison, the exact value is $C^e(101, 0) = \$13.7137$.

Vega $\mathcal{V}$ (1 of 2)

Vega is the partial derivative with respect to volatility σ.

$$\begin{aligned}
C^e &= S\,\Phi(w) - K\,e^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) \\
\frac{\partial C^e}{\partial \sigma} &= S\,\phi(w)\frac{\partial w}{\partial \sigma} - K\,e^{-r(T-t)}\phi\left(w - \sigma\sqrt{T-t}\right)\left[\frac{\partial w}{\partial \sigma} - \sqrt{T-t}\right] \\
&= S\,\phi(w)\frac{\partial w}{\partial \sigma} - S\,\phi(w)\left[\frac{\partial w}{\partial \sigma} - \sqrt{T-t}\right] \\
&= \frac{S\sqrt{T-t}}{\sqrt{2\pi}}e^{-w^2/2} > 0
\end{aligned}$$

Vega $\mathcal{V}$ (2 of 2)

According to the Put-Call Parity formula:

$$\begin{aligned}
P^e + S &= C^e + K\,e^{-r(T-t)} \\
\frac{\partial}{\partial\sigma}\left[P^e + S\right] &= \frac{\partial}{\partial\sigma}\left[C^e + K\,e^{-r(T-t)}\right] \\
\frac{\partial P^e}{\partial\sigma} &= \frac{\partial C^e}{\partial\sigma} \\
\frac{\partial P^e}{\partial\sigma} &= \frac{S\sqrt{T-t}}{\sqrt{2\pi}}e^{-w^2/2}
\end{aligned}$$

Remark: vega is identical for puts and calls.

Example (1 of 2)

Consider a three-month European put option on a non-dividend-paying stock whose current value is \$200 and whose volatility is 30%. The option has a strike price of \$195 and the risk-free interest rate is 6.25%.

1. Find the vega of the option.
2. If the volatility of the stock increases to 31%, approximate the change in the value of the put.

Example (2 of 2)

$$
\begin{aligned}
S &= 200 \quad \sigma = 0.30 \quad T = 3/12 \\
K &= 195 \quad r = 0.0625 \quad t = 0
\end{aligned}
$$

Using these values

$$
\begin{aligned}
w &= \frac{\ln(S/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}} \\
\mathcal{V} &= \frac{S\sqrt{T - t}}{\sqrt{2\pi}} e^{-w^2/2}
\end{aligned}
$$

Example (2 of 2)

$$\begin{aligned} S &= 200 \quad \sigma = 0.30 \quad T = 3/12 \\ K &= 195 \quad r = 0.0625 \quad t = 0 \end{aligned}$$

Using these values

$$\begin{aligned} w &= \frac{\ln(S/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}} \approx 0.347952 \\ \mathcal{V} &= \frac{S\sqrt{T - t}}{\sqrt{2\pi}} e^{-w^2/2} \approx 37.5509 \end{aligned}$$

Example (2 of 2)

$$\begin{aligned} S &= 200 \quad \sigma = 0.30 \quad T = 3/12 \\ K &= 195 \quad r = 0.0625 \quad t = 0 \end{aligned}$$

Using these values

$$\begin{aligned} w &= \frac{\ln(S/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}} \approx 0.347952 \\ \mathcal{V} &= \frac{S\sqrt{T - t}}{\sqrt{2\pi}} e^{-w^2/2} \approx 37.5509 \end{aligned}$$

Using the linear approximation,

$$dP = \mathcal{V}\, d\sigma = (37.5509)(0.01) = 0.375509.$$

Rho ρ (1 of 2)

Rho is the partial derivative with respect to the risk-free interest rate r.

$$
\begin{aligned}
C^e &= S\,\Phi\left(w\right) - K\,e^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) \\
\frac{\partial C^e}{\partial r} &= S\,\phi\left(w\right)\frac{\partial w}{\partial r} + K(T-t)e^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) \\
&\quad - K\,e^{-r(T-t)}\phi\left(w - \sigma\sqrt{T-t}\right)\frac{\partial w}{\partial r} \\
&= K(T-t)e^{-r(T-t)}\Phi\left(w - \sigma\sqrt{T-t}\right) > 0
\end{aligned}
$$

Rho ρ (2 of 2)

Starting with the Put-Call Parity formula:

$$\begin{aligned}
P^e + S &= C^e + K\,e^{-r(T-t)} \\
\frac{\partial}{\partial r}\left[P^e + S\right] &= \frac{\partial}{\partial r}\left[C^e + K\,e^{-r(T-t)}\right] \\
\frac{\partial P^e}{\partial r} &= \frac{\partial C^e}{\partial r} - K(T-t)e^{-r(T-t)} \\
\frac{\partial P^e}{\partial r} &= -K(T-t)e^{-r(T-t)}\Phi\left(\sigma\sqrt{T-t} - w\right) < 0
\end{aligned}$$

Example (1 of 2)

Consider a three-month European put option on a stock whose current value is \$200 and whose volatility is 30%. The option has a strike price of \$195 and the risk-free interest rate is 6.25%.

1. Find the rho of the option.
2. If the interest rate increases to 7.00%, approximate the change in the value of the put.

Example (2 of 2)

$$
\begin{aligned}
S &= 200 \quad \sigma = 0.30 \quad T = 3/12 \\
K &= 195 \quad r = 0.0625 \quad t = 0
\end{aligned}
$$

Using these values

$$
\begin{aligned}
w &= \frac{\ln(S/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}} \\
\rho &= -K(T - t)e^{-r(T-t)}\Phi\left(\sigma\sqrt{T - t} - w\right)
\end{aligned}
$$

Example (2 of 2)

$$\begin{aligned} S &= 200 \quad \sigma = 0.30 \quad T = 3/12 \\ K &= 195 \quad r = 0.0625 \quad t = 0 \end{aligned}$$

Using these values

$$\begin{aligned} w &= \frac{\ln(S/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}} \approx 0.347952 \\ \rho &= -K(T - t)e^{-r(T-t)}\Phi\left(\sigma\sqrt{T - t} - w\right) \approx -20.2315 \end{aligned}$$

Example (2 of 2)

$$\begin{aligned} S &= 200 \quad \sigma = 0.30 \quad T = 3/12 \\ K &= 195 \quad r = 0.0625 \quad t = 0 \end{aligned}$$

Using these values

$$\begin{aligned} w &= \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \approx 0.347952 \\ \rho &= -K(T-t)e^{-r(T-t)}\Phi\left(\sigma\sqrt{T-t} - w\right) \approx -20.2315 \end{aligned}$$

Using the linear approximation,

$$dP = \rho\, dr = (-20.2315)(0.0075) = -0.151737.$$

Homework

- Read Chapter 9.
- Exercises: 1–11.

Credits

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