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Blackjack Ace Prediction

It's every quant's favorite card game. Now, isn't it time you understood it?

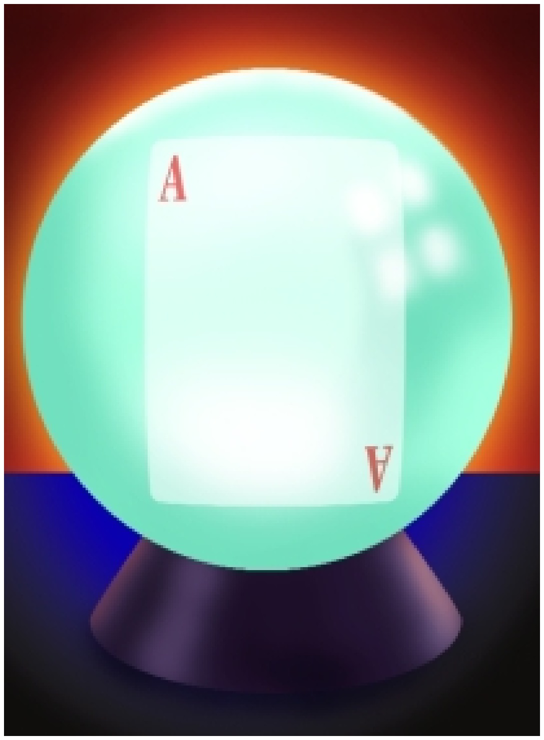
Ed Thorp said, "The big thrill," "came from learning things nobody else in the world had ever known."[1] Edward Oakley Thorp was a 28-year-old Assistant Professor at New Mexico State University when he came up with the idea of Ace Prediction: "I believe that I began to think in detail about the non-randomness of human shuffling in 1961 and 1962. My initial thoughts were that it could very substantially affect the odds of many games.

"This was confirmed by the subsequent work I did. I had a two-pronged attack: build mathematical models to approximate real shuffling, and do empirical studies of real shuffling. While doing this, I wanted a simple, practical method for exploiting this and the idea of ace locating, using neighboring cards, occurred to me. Why aces? Because an ace is the best card for the player to get as one of his initial two cards at blackjack.

"I tried it out at home and it worked well. I didn't focus on using it at the casinos because many other projects with higher priority were going on in my life at the same time."

Among Thorp's "other projects" were inventing with Claude Shannon (1916–2001) the world's first wearable computer to successfully predict roulette outcomes in Las Vegas, and writing the world's best-selling gambling book, *Beat the Dealer*,[2] which contained the first mathematical system ever discovered for beating a major casino game—card counting at blackjack.

While Thorp's book made the *New York Times* bestseller list, his Ace Prediction theory remained the closely guarded secret of a handful of high-stakes professional blackjack players for more than 20 years.



How predictable are casino shuffles?

Professional blackjack players analyze the predictability of casino shuffles before trying to predict aces in the casino. Armed with information about the way cards move around in a particular shuffle, they are much more likely to win than the average player.

Here is one example. Assume we know the exact order of a deck of 52 cards prior to it being shuffled. The probability that the deck is in any specific post-shuffle order i is denoted by p_i ($p_i = 1$ for a single i).

Shannon's formula for *informational entropy*[3] gives the amount of *uncertainty* U associated with this situation:

$$U = -\sum_{i=1}^{n!} p_i \log_2 p_i$$

Where,

$$n = 52$$
$$p_i = 1$$

Thus,

$$U = -\sum_{i=1}^{52!} (1)\log_2(1) = 0$$

As we might expect, since we know the exact order of the deck, there is no *uncertainty* at all. Conversely, the amount of *information* (in *bits*) is given by:

$$I = \log_2(n!) - U$$

Where,

$$n = 52$$
$$U = 0$$
$$I = \log_2(52!) - 0 = 225.58 \text{ bits}$$

Starting with a deck in known order, each successive riffle reduces the percentage of known information to the levels shown in Table 1:[4]

After the first riffle, $\log_2 2^{52} = 52$ bits of information about deck order are *destroyed* (23.05%) and 173.58 bits *remain* (76.95%). Reductions of similar magnitude occur for the second and third riffles. After the fourth riffle, only 12% of the original information remains. As information is lost, uncertainty increases. After ten riffles, $I = 0$ bits and $U = \log_2(52!)$ $= 225.58$ ($p_i = 1/n!$ for all i).

Thus, professional gamblers limit themselves to predicting aces only in those games where the dealer makes a maximum of three riffle shuffles. With a simple three-riffle shuffle, enough information remains to make the post-shuffle deck order *reasonably* predictable. Four or more riffles are too unpredictable.

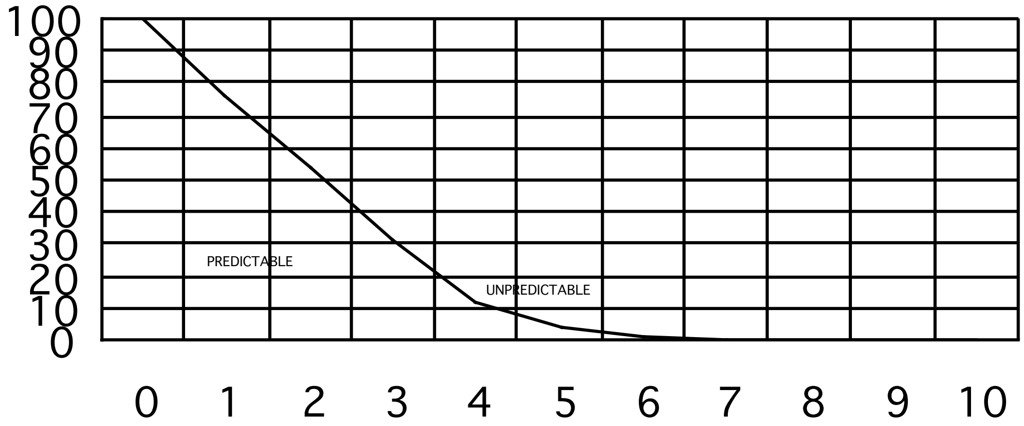


Figure 1: Information Loss in Card Shuffling

Table 1: Information Loss in Card Shuffling

Riffles	*Info %*
0	100
1	76.95
2	53.90
3	30.98
4	12.09
5	3.52
6	0.92
7	0.23
8	0.06
9	0.01
10	0.00

Predicting aces at the gaming table

Predicting aces at the blackjack table is a lot easier than the theory makes it look.

In this example, you have already seen the first *four* rounds of play in a six-deck game. Having counted the cards as they were dealt, you know there are exactly 49 cards in the discard tray.

By carefully analyzing the shuffle, you know most of the cards in the *next* round are *very likely* to be in the fourth half-deck from the top after the shuffle. On the *fifth* round, an ace appears:

You note the “key” card that will be *under* the ace when the dealer scoops up the cards. After ten more rounds, the dealer begins to shuffle the cards.

You *eyeball* the shuffle closely and track the ace to its final location. You are offered the cut card, and cut to bring the ace straight to the top of the six decks. On the *first* round of the new shoe. . .

. . . the key card appears! You count the number of cards following the key card (two) and predict the ace will appear on box $4 - 2 = 2$ in the next round. You make a large wager on box 2 . . .

. . . and the ace falls on box 2! This is an extremely elegant and incredibly powerful technique.

The lucrative profits of skill

Let us compare the expected results of a *highly skilled* professional ace predictor with those of an ordinary *unskilled* player—that is, someone who predicts aces merely by guessing.

For instance, in our six-deck game, the unskilled “guesser” finds that, on average, he predicts successfully one ace in every thirteen attempts: $\mu = 13 \times 24/(24 + 288) = 1$, yielding an overall “success rate” of 0.07.

The skilled ace tracker, on the other hand, predicts an average of almost *two* aces in thirteen attempts: $\mu = 13 \times 42/(24 + 288) = 1.75$, with a hit rate of around 0.13. In other words, the skilled player is nearly *twice as good* at predicting aces as the unskilled player (Table 2):

Table 2 reveals the skilled predictor makes *one or more* correct predictions (for every thirteen attempts) 85% of the time while the guesser manages the same only 65% of the time. However, consider how *slight* the difference is between the

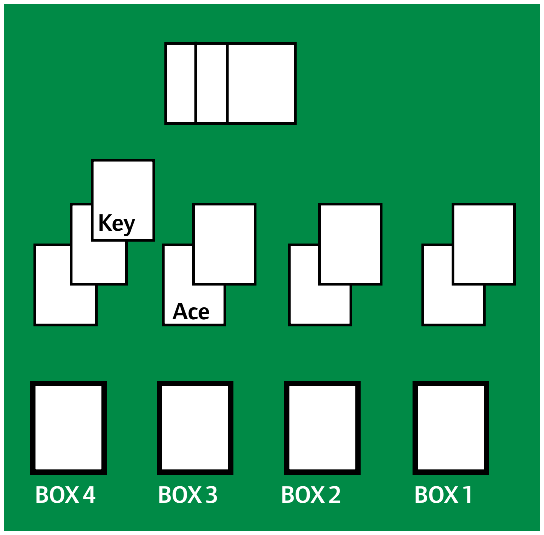


Figure 2: An Ace in A Trackable Position

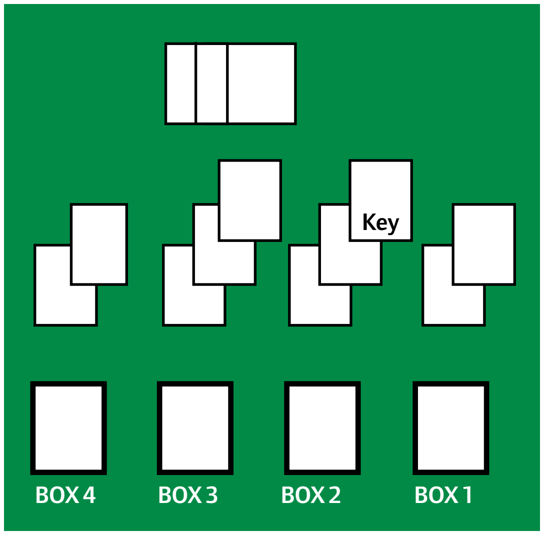


Figure 3: The Key Card Appears

expert and the guesser. In the above example, the expert tracker predicts just *six* more aces in every hundred than the guesser, but because the ace is the most important card in blackjack these six hands give the expert a significant *advantage* over the house – the source of the expert’s lucrative profits.

The Gambler King

The two stunning blondes, one on each arm, immediately reminded me of Frank Sinatra’s

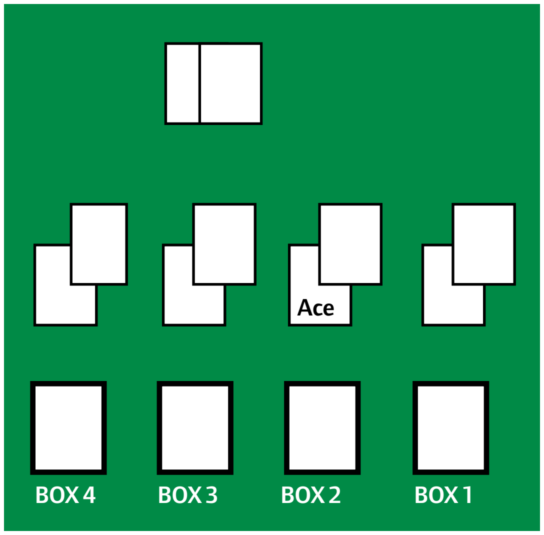


Figure 4: The Ace Falls on The Predicted Box

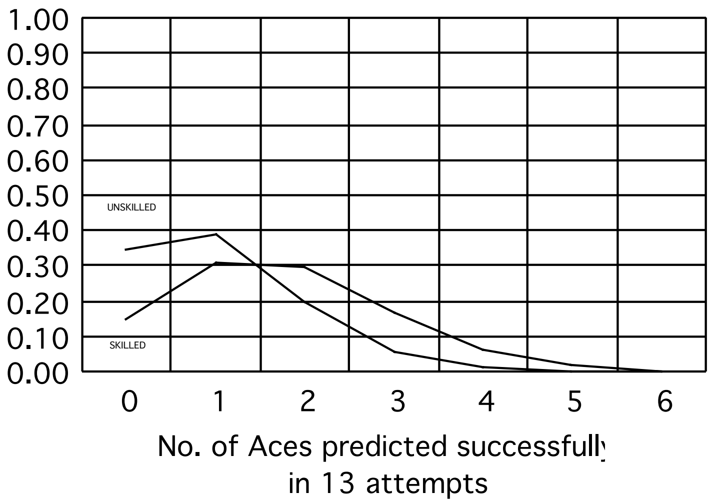


Figure 5: The Advantage of Skill

famous gag to a pit boss at the *Sands* in Las Vegas. Flanked by two lovely ladies, Frank had asked: “What do you think of the cufflinks?”

“The Gambler King” was at least sixty, overweight, and not particularly handsome. He was wearing sunglasses, a white Armani suit, a white silk shirt fastened at the neck by a silver stud, and a pair of spotless white shoes. His long silver-gray hair was pulled back tightly into a ponytail. His “cufflinks,” about thirty-five years younger than him, had never been inside a casino before. They were too frightened to gamble, like fawns caught in car headlights.

The two women started to giggle like schoolgirls. As predicted, the first card out was the ace, the dealer got a 5 and, as our second card hit the felt, they let out a little squeal. It was another ace!

Table 2: Skilled vs. Unskilled Predictors

Aces	Skilled	Unskilled
0	0.15	0.35
1	0.31	0.39
2	0.29	0.19
3	0.17	0.06
4	0.06	0.01
5	0.02	0.00
6	0.00	0.00
Totals	1.00	1.00
≥1	0.85	0.65

I was expecting an ace on the next hand so I changed up 1,000 French francs (about $200) and placed my bet on first base. As I did, the King casually tossed 5,000 French francs on to the blackjack table. The dealer quickly counted the notes then, in one slick motion, lifted a tube of gold chips from the tray; spread, counted, and restacked them, before sliding them back across the green baize.

“All” snapped the King, pointing to first base. The dealer pushed the King’s towering pile of chips alongside my single, 1,000-franc chip. As the dealer swished out the cards, the two women started to giggle like schoolgirls. As predicted, the first card out was the ace, the dealer got a 5 and, as our second card hit the felt, they let out a little squeal. It was another ace!

Calmly, the King threw another 5,000 francs on to the table, and, said: “Split.” There was a murmur of excitement. A small crowd, sensing something unusual, started to gather round. The dealer separated the two aces, before arranging another enormous stack of chips next to the second ace, along with another of my 1,000-franc chips. The Gambler King now stood to win or lose 10,000 francs on this one hand.

The cards flashed again. Bang, bang! Two jacks! Double vingt et un! The dealer drew a 10 and a 7 to bust with 22. The audience burst into spontaneous applause.

Waving theatrically to his gaggle of admirers, a little smirk appeared on the King’s face that said: “It was nothing, really.” As he hurriedly pocketed his 20,000 francs he winked at me and whispered: “*C’est assez pour ce soir*” (that’s enough for tonight!).

REFERENCES

1 O’NEIL, PAUL, “The Professor Who Breaks the Bank,” Life, Chicago, Illinois: Time, Inc., March 27, 1964, p. 84.
2 THORP, EDWARD O., *Beat the Dealer: A Winning Strategy for the Game of Twenty-One*, New York: Blaisdell Publishing Company, 1st ed., 1962.
3 EPSTEIN, RICHARD A., *The Theory of Gambling and Statistical Logic*. San Diego, California: Academic Press, 1977, p. 167.
4 TREFETHAN, LLOYD N., AND LLOYD M. TREFETHAN, “How many shuffles to randomize a deck of cards?” *Proceedings of the Royal Society of London, Series A*, 1999.