The Korn-Kreer-Lenssen Model as an Alternative for Option Pricing

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Abstract: The first purpose of this paper is to illustrate how the Korn-Kreer-Lenssen model can be implemented in pricing European vanilla options and to analyze the accuracy of this model. The Korn-Kreer-Lenssen model assumes that the underlying stock price follows a linear birth-death process rather than a geometric Brownian motion. The second purpose of this paper is to derive two closed-form solutions for pricing American digital options in the Korn-Kreer-Lenssen's framework, by using the birth-death process theory, as well as the probability distribution of the first passage time of the underlying stock process.

1. Introduction

The type of stochastic process, which determines the movement of the stock price, is of prime importance in option valuation. Traditionally, the movement of stock price is described to follow a geometric Brownian motion. This implies that changes in the log prices (log returns) over fixed intervals of time are i.i.d. and Gaussian random variables.

However, the geometric Brownian motion model has been questioned by a number of researchers. For example, Mandelbrot (1963) and Fama (1963, 1965) observed that the distributions of log returns are not normal distributions rather they have very thick tails particularly over short periods of time such as over one day. As a result of his observations, Mandelbrot (1963) proposed that the thick tailed alternative models would be more appropriate in describing the movement of stock price.

Cox and Ross (1976) proposed that the birth-death process would be another kind of alternative stock price model compared to the geometric Brownian motion. This model serves to explain situations when jumps in stock prices occur and resolves a number of important problems such as payouts and bankruptcies, which would be intractable for the lognormal model. For example, stock prices have a positive probability of hitting and remaining at zero, an event that corresponds to the bankruptcy of a firm. The birth−death process is able to capture the absorbing barrier at the origin. The limitation of Cox and Ross' model (1976) is that it assumes that jumps in stock prices would not occur more than once within a short period of time.

Korn, Kreer and Lenssen (1998) extended the Cox and Ross' (1976) model to a more general jump model with state dependent jump intensities. This model is able to overcome the limitation of Cox and Ross' model, by capturing situations when more frequent jumps in stock prices would occur. Korn, Kreer and Lenssen investigated a possible approach to price European vanilla options when the underlying stock price follows a linear birth-death process. They derived explicit pricing formulas for European vanilla options by arbitrage arguments.

This paper illustrates how the Korn-Kreer-Lenssen model can be implemented in pricing European vanilla options. A numerical example is used to demonstrate the application and accuracy of this model. We also derive two closed-form solutions for American digital options in the Korn-Kreer-Lenssen's framework.

The structure of this paper is as follows. Section 2 presents a linear birth-death process for stock prices. In section 3, we derive two explicit pricing formulas for American digital options. A numerical example is presented in section 4 to illustrate the implementation and testing of the Korn-Kreer-Lenssen model for European vanilla options. Conclusions are drawn in section 5.

2. The Stock Price Model

We model the movement of the stock price $S = S_t$ by a jump process in continuous time t with zero drift. More formally, given some $T > 0$, the movement of the stock price S_t in continuous time $t \in [0, T]$ can be described as follows

$$dS_t = dN_t(1) - dN_t(2), \tag{2.1}$$

where $N_t(1)$ and $N_t(2)$ are jump processes on the time interval $[0, T]$ with jump size 1 and intensities λS_t and ηS_t, respectively. It is assumed that the jump processes are defined on the probability space (Ω, F, P) where the history information is given by the filtration $\{F_t\}_{t\in[0,T]}$ generated by $N_t(1)$ and $N_t(2)$. Also we assume $F = F_T$. Obviously, the two processes,

$$N_t(1) - \int_0^t \lambda S_s ds \text{ and } N_t(2) - \int_0^t \eta S_s ds,$$

are martingales with respect to the filtration $\{F_t\}_{t\in[0,T]}$.

Taking the initial value S_0 to be a non−negative integer we may restrict the stock price $S = S_t$ to a non−negative integer value only. Note that $\lambda \cdot 0 = 0$, $\eta \cdot 0 = 0$ guarantee that the stock price will never become negative, that is, $S \in \{0, 1, 2, \ldots\}$. In case that at some point in time $t^* \in [0, T]$ the stock price become zero, the choice of the transition rates λ and η guarantees that S_t will be zero for all later times $t \in [t^*, T]$. That is to say, the model allows for cases of bankruptcy. See figures at p.651 in Korn, Kreer and Lessen (1998) for typical sample paths of linear birth-death processes.

We denote the probability distribution of stock price by $p_{n,j}(t, t_0) = P(S_t = j|S_{t_0} = n)$, which indicates the probability of stock price being equal to j at time t conditional on the initial value $S_{t_0} = n$ at a previous time t_0. By the time homogeneous of S_t, we can further have $p_{n,j}(t, t_0) = p_{n,j}(t - t_0, 0) \equiv p_{n,j}(t - t_0)$. These probabilities satisfy the well−known forward birth-death equations (keeping both t_0 and $S_{t_0} = n$ fixed, and writing $\tau = t - t_0$)

$$\begin{aligned}
&\frac{\partial}{\partial \tau} p_{n,0} = \eta p_{n,1}, \\
&\frac{\partial}{\partial \tau} p_{n,j} = \lambda(j-1)p_{n,j-1} - (\lambda+\eta) j p_{n,j} + \eta(j+1)p_{n,j+1}, j \geq 1, \\
&p_{n,j}(0) = \delta_{nj},
\end{aligned} \tag{2.2}$$

where $p_{n,-1}(\tau) = 0$.

The probabilities also satisfy the system of backward birth-death equations (keeping both t and $S_t = j$ fixed, and writing $\tau = t - t_0$)

$$\begin{aligned}
&\frac{\partial}{\partial \tau} p_{0,j} = 0, \\
&\frac{\partial}{\partial \tau} p_{n,j} = \eta n p_{n,n-1} - (\lambda+\eta) n p_{n,j} + \lambda(n+1)p_{n+1,j}, j \geq 1.
\end{aligned} \tag{2.3}$$

By solving the infinite set of coupled ordinary differential equations (2.2), the probability distribution of stock price $p_{n,j}$, is obtained.

The basic assumptions concerning the financial market in this paper are stated as follows.

- Trading is continuous in $[0, T]$ and takes place in a liquid frictionless market.
- Short−selling with full use of proceeds is allowed.
- In addition to the stock S, the market trades a European style Low Exercise Price Option, F^*, with strike 1 and maturity T. We may assume this LEPO F^* to be a put option with strike 1.
- The rate r, at which individuals can borrow and lend freely, is constant.

3. Closed Form Solutions for Pricing American Digital Options

Exotic options are a generic name given to derivative securities that have more complex cash flow structures than standard options. The main motivation for trading exotic options is that they permit a much more precise articulation of views on future market behavior than those offered by standard options. The trading of exotic options requires acute timing skills in hedging and the use of options to manage volatility risk.

The simplest exotic option is binary or digital option. A digital option is a contingent claim on some underlying asset or commodity that has an all− or− nothing payoff. A digital call option has a payoff

$$B(S_T) = \begin{cases} 1 & \text{if } S_T \geq K, \\ 0 & \text{if } S_T < K, \end{cases} \tag{3.1}$$

and a digital put has payoff $1 - F(S_T)$.

In this section, we first review some results for European vanilla options given by Korn, Kreer and Lessen (1998), then we derive the new results for American digital options.

For European vanilla options when the underlying stock price follows a jump process, basically there are two approaches to derive option pricing partial differential-difference equations. One approach is the hedging argument as in Cox and Ross (1975b) or Korn, Kreer and Lenssen (1998) p.657 to p.659. The other approach is the Capital Asset Pricing Model (CAPM) argument as in Merton (1992) p.318 to p.319, see Appendix A for more details.

The no arbitrage argument for pricing contingent claim B payable at time T states that the price f_t of this contingent claim at time $t < T$ must be given as

$$f_t = \hat{E}(e^{-r(T-t)}B|F_t), \tag{3.2}$$

where $\hat{E}$ is the expectation with respect to some equivalent martingale measure $\hat{P}$ is a martingale with respect to the equivalent martingale measure. See e.g. Harrison and Pliska (1981) for the presentation of the relevant arguments.

The existence of equivalent martingale measure is stated as follows.

Proposition 3.1 A probability measure $\hat{P}$ associated to a linear birth-death process with transition rates $\hat{\lambda}$ and $\hat{\eta}$ is an equivalent martingale measure for the security price if and only if we have

$$\hat{\lambda} = \hat{\eta} + r. \tag{3.3}$$

Proposition 3.1 indicates that in a risk neutral world (i.e. the discounted stock price is a martingale) the transition rates for up–jumps are higher than those for down– jumps.

We will further refer transition rates satisfying equation (3.3) as risk adjusted rates.

The following lemma will enable us to provide not only explicit representations for the price of European options, but also a sufficient condition for the uniqueness of equivalent martingale measure.

Lemma 3.2 The solution of the forward birth-death equation (2.2), with the risk adjusted rates $\hat{\lambda}$ and $\hat{\eta}$ satisfying (3.3), subject to the initial condition $\hat{P}_{n,n}(t,t) = 1$ and $\hat{P}_{n,j}(t,t) = 0$ for all other $j \neq n$, is given by

$$\begin{aligned} \hat{P}_{n,0}(s,t) &= \alpha^n, \\ \hat{P}_{n,j}(s,t) &= \sum_{i=0}^{\min(n,j)} \binom{n}{i}\binom{j+n-i-1}{n-1} \alpha^{n-i}\beta^{j-i}(1-\alpha-\beta)^i, \\ & \quad j \geq 1, t < s, \end{aligned} \tag{3.4}$$

where

$$\begin{aligned} \alpha &= \frac{\hat{\eta}\left(e^{r(s-t)}-1\right)}{\hat{\eta}\left(e^{r(s-t)}-1\right)+re^{r(s-t)}}, \\ \beta &= \frac{(\hat{\eta}+r)\left(e^{r(s-t)}-1\right)}{\hat{\eta}\left(e^{r(s-t)}-1\right)+re^{r(s-t)}}. \end{aligned} \tag{3.5}$$

Proposition 3.3 Let the market price F_t^* at time $t < T$ of the LEPO put with strike 1 and maturity T satisfy the regular condition $e^{r(T-t)}F_t^* \leq 1$. Then the positive parameter $\hat{\eta}$ is uniquely determined at time t by the market price of the LEPO put as solution of

$$F_t^* = e^{-r(T-t)}\alpha^n, \tag{3.6}$$

where α is defined in equation (3.5) and $\alpha \in (0,1)$. Thus under the above regular condition, an equivalent martingale measure $\hat{P}$ can be uniquely determined.

Theorem 3.4 The fair price $f_{t,n}$ of a European call option at time t, when the stock price S_t is equal to n, is given by

$$\begin{aligned} call(T,t,S_t=n) &= e^{-r(T-t)}\sum_{j=K+1}^{\infty}(j-K)\sum_{i=0}^{\min(n,j)}\binom{n}{i}\binom{j+n-i-1}{n-1} \\ & \quad \times \alpha^{n-i}\beta^{j-i}(-\alpha-\beta)^i. \end{aligned} \tag{3.7}$$

While in the case of a European put option, we have the following pricing formula

$$\begin{aligned} put(T,t,S_t=n) &= e^{-r(T-t)}\sum_{j=0}^{K-1}(K-j)\sum_{i=0}^{\min(n,j)}\binom{n}{i}\binom{j+n-i-1}{n-1} \\ & \quad \times \alpha^{n-i}\beta^{j-i}(1-\alpha-\beta)^i. \end{aligned} \tag{3.8}$$

Remark: The regular condition in Proposition 3.3 for the uniqueness of $\hat{\eta}$ is sufficient but not necessary. As long as $\hat{\eta}$ can be uniquely implied from market prices of some traded options which have the same underlying as that of the option being priced, Theorem 3.4 would still be valid.

For detailed proofs of Proposition 3.1, Lemma 3.2, Proposition 3.3 and Theorem 3.4, see Korn, Kreer and Lessen (1998).

From now on, we discuss pricing American digital options. The American digital call option has a payoff of one dollar if the underlying asset reaches the strike price K before or at the expiration date T. This introduces additional time option to the pricing problem. According to the general principles, the fair value of the American digital call option at time 0 is given by

$$f_{0,n_0} = e^{-rT}\hat{P}(T_K \leq T|S_0 = n_0). \tag{3.9}$$

In order to give an explicit pricing formula, we need to investigate the probability distribution of the first passage time T_K of the stock process crossing the strike price boundary K.

To investigate T_K, consider a new process in which the transition probability is modified by making K an absorbing state. Equation (2.2) is unaltered for $j = 0, 1, \ldots, K-2$, and for $j = K-1, K$ the equation is replaced by

$$\begin{aligned} \frac{\partial}{\partial t}p_{n,K-1}(t) &= \lambda(K-2)p_{n,K-2}(t) - (\lambda+\eta)(K-1)p_{n,K-1}(t), \\ \frac{\partial}{\partial t}p_{n,K}(t) &= \lambda(K-1)p_{n,K-1}(t). \end{aligned} \tag{3.10}$$

However, in this situation, the backward equations are rather more convenient than the forward equations.

The forward equations are obtained by fixing $S_0 = n_0$ and relating the states of the process at time 0, t and $t + \Delta t$. The backward equations are obtained by fixing $S_t = K$

and relating the states of the process at time $-\Delta t$, 0 and t. For the process with an absorbing state K, we have

$$P_{j,K}(t+\Delta t) = P_{j,K}(t)\{1-(\lambda+\eta)j\Delta t\} + P_{j-1,K}(t)\eta j\Delta t + P_{j+1,K}(t)\lambda j\Delta t + o(\Delta t),$$

leading to

$$\frac{\partial}{\partial t}p_{j,K}(t) = -(\lambda+\eta)jp_{j,K}(t) + \eta jp_{j-1,K}(t) + \lambda jp_{j+1,K}(t), \quad 0 \le j \le K-1, \tag{3.11}$$

with $P_{K,K}(t) = 1$, $P_{j,K}(0) = 0$.

Noticing that $\hat{P}(T_K \le t|S_0 = n_0) = P_{n0}, K(t)$, an explicit pricing formula for American digital call option can be given from (3.9) as long as we obtain the solution of (3.11).

Denote

$$Q_n(t) = \frac{dP_{n,K}(t)}{dt}. \tag{3.12}$$

Applying (3.12) to the backward equation (3.11), we have

$$\begin{aligned} &Q_n'(t) = -(\lambda+\eta)nQ_n(t) + \eta nQ_{n-1}(t) + \lambda nQ_{n+1}(t),\ 0 \le n \le K-1,\\ &Q_K(t) = 0,\\ &Q_n(0) = \delta_{n,K-1}\lambda(K-1). \end{aligned} \tag{3.13}$$

The solution of (3.13) is derived from the following theorem.

Theorem 3.5 The solution of equation (3.13) is given by

$$\begin{aligned} Q_n(t) = &-\frac{(-1)^{n-1}\eta r^2}{(n-1)!}\int_0^t \Bigg[Q_{K-1}(s)e^{-r(t-s)}\frac{\left\{\lambda\left(1-e^{-r(t-s)}\right)\right\}^{K-n}}{\left\{\lambda-\eta e^{-r(t-s)}\right\}^{K+1}}\\ &\times\left\{\eta-\lambda e^{-r(t-s)}\right\}^{n-1}\\ &\times\sum_{i=0}^{n-1}\binom{n-1}{i}\frac{\Gamma(K+i+1)}{\Gamma(K-n+i+1)}\\ &\times\left\{\frac{-\lambda\eta\left(1-e^{-r(t-s)}\right)^2}{\left(\eta-\lambda e^{-r(t-s)}\right)\left(\lambda-\eta e^{-r(t-s)}\right)}\right\}^i\Bigg]ds\\ &+\frac{(-1)^{n-1}\lambda r^2}{(n-1)!}e^{-rt}\frac{\{\lambda(1-e^{-rt})\}^{K-n-1}}{\{\lambda-\eta e^{-rt}\}^K}\left\{\eta-\lambda e^{-rt}\right\}^{n-1}\\ &\times\sum_{i=0}^{n-1}\binom{n-1}{i}\frac{\Gamma(K+i)}{\Gamma(K-n+i)}\left\{\frac{-\lambda\eta\left(1-e^{-rt}\right)}{(\eta-\lambda e^{-rt})(\lambda-\eta e^{-rt})}\right\}^i \end{aligned} \tag{3.14}$$

Proof: We denote the generating function

$$Q(z,t) = \sum_{n=1}^{K-1} Q_n(t)z^{n-1}. \tag{3.15}$$

From the initial conditions in (3.13), we have

$$Q(z,0) = \lambda(K-1)z^{K-2}. \tag{3.16}$$

Multiplying the equations in (3.13) by z^{n-1} and summing over n we have the partial differential equation for Q

$$-\frac{\partial Q}{\partial t} + \{\eta z^2 - (\lambda+\eta)z + \lambda\}\frac{\partial Q}{\partial z} = (\lambda+\eta-2\eta z)Q + \eta KQ_{K-1}(t)Z^{K-1}. \tag{3.17}$$

The solution of (3.17) is obtained as

$$\begin{aligned} Q(z,t) = &-K\eta r^2\int_0^t Q_{K-1}(s)e^{-r(t-s)}\frac{\{\lambda(1-e^{-r(t-s)}) - z(\eta-\lambda e^{-r(t-s)})\}^{K-1}}{\{(\lambda-\eta e^{-r(t-s)}) - \eta z(1-e^{-r(t-s)})\}^{K+1}}ds\\ &+(K-1)\lambda r^2 e^{-rt}\frac{\{\lambda(1-e^{-rt}) - z(\eta-\lambda e^{-rt})\}^{K-2}}{\{(\lambda-\eta e^{-rt}) - \eta z(1-e^{-rt})\}^{K}}. \end{aligned} \tag{3.18}$$

From (3.15) we have

$$Q_n(t) = \left\{\frac{1}{(n-1)!}\frac{\partial^{n-1}Q(z,t)}{\partial z^{n-1}}\right\}_{z=0}. \tag{3.19}$$

Combining (3.18) and (3.19), we get (3.14).

In the following theorem, we give the first closed form solution for pricing American digital options when the underlying stock dynamic follows a linear birth-death process.

Theorem 3.6 Let the market price F_t^* at time $t < T$ of the LEPO put with strike 1 and maturity T satisfy the regular condition $e^{r(T-t)}F_t^* \le 1$. Then the fair price f_{0,n_0} of an American digital call option at time 0 when the stock price S_0 is equal to n_0 is given by

$$f_{0,n_0} = f(0,T,S_0 = n_0) = e^{-rT}\int_0^T Q_{n_0}(t)dt, \tag{3.20}$$

where $Q_{n_0}(t)$ has an expression from (3.14).

Proof. Since K is the absorbing barrier of the underlying linear birth-death process, it is easy to see

$$\hat{P}(T_K \le t|S_0 = n_0) = P_{n_0,K}(t). \tag{3.21}$$

Combining (3.9), (3.21) and (3.12), we prove the theorem.

To investigate T_K in more detail, we define the following polynomials:

$$\begin{aligned} &L_0(x) \equiv 1,\\ &L_1(x) = x+\lambda,\\ &L_n(x) = (x+\lambda+\eta)L_{n-1}(x) - \lambda\eta L_{n-2}(x)(n \ge 2). \end{aligned} \tag{3.22}$$

In the following Theorem 3.7, we get a more explicit expression for the distribution density function of T_K.

Theorem 3.7 Suppose $S_0 = n_0$, then T_K, the first arriving time at state K of the linear birth-death process $\{S_t, t \in [0, T]\}$, has the distribution density function

$$Q_{n_0}(t) = \sum_{j=1}^{K} \frac{\lambda^{K-n_0} L_{n_0}(-\mu_j^{(K)})}{L_K(-\mu_j^{(K)})} e^{-\mu_j^{(K)} t}, \tag{3.23}$$

where $-\mu_j^K$ is the zero point of $L_K(x)$.

Proof. See Wang Zikun and Yang Xiangqun (1992) Theorem 1' at p. 169.

In the following theorem, we give the second closed form solution for pricing American digital options when the underlying stock dynamic follows a linear birth-death process.

Theorem 3.8 Let the market price F_t^* at time $t < T$ of the LEPO put with strike 1 and maturity T satisfy the regular condition $e^{r(T-t)} F_t^* \leq 1$. Then the fair price f_{0,n_0} of an American digital call option at time 0 is given by

$$f_{0,n_0} = e^{-rT} \int_0^T Q_{n_0}(t) dt = e^{-rT} \sum_{j=1}^{K} \frac{\lambda^{K-n_0} L_{n_0}(-\mu_j^{(K)})}{L_K(-\mu_j^{(K)})} \frac{1 - e^{-\mu_j^{(K)} T}}{\mu_j^{(K)}} \tag{3.24}$$

Proof. Combining (3.9) and (3.23), we get (3.24).

As we can see, the pricing formula (3.24) for American digital call option is much simpler than the pricing formula (3.20) in terms of numerical calculations.

4. A Numerical Example

In this section, we give an illustrative example on how to calibrate the Korn-Kreer-Lenssen formula (3.8) to price a European put option where the underlying stock price follows a linear birth-death process. In addition, the accuracy of this model in pricing the option is analyzed.

The following data is assumed and used simply for illustration purpose. Suppose we have European put option data of a company on August 28, 2002. At the time of observation, suppose stock price of the company is 1.58$ and market prices of the European put options of the company are in Table 1.

Since the stock price process is a birth-death process with non-negative integer state space, more accurately, we use a finer unit, for example, a dime unit for stock price in the pricing formula (3.8). Hence the current stock price is assumed to be 15.

In the formula (3.8),

n = 15
K = strike price in Table 1
t = August 28, 2002 —— current date
T = maturity date in Table 1
T-t = business days between current date and maturity date/365
r = 4.5% ——— prime rate.

α and β in (3.8) can be obtained from formula (3.5) as long as we have an estimation for parameter $\hat{\eta}$.

TABLE I:

Maturity date	Strike price in dollar unit	Average of bid and ask market price
October 2, 2002	1.5	0.225
October 2, 2002	2.0	0.55
October 2, 2002	2.5	0.975
October 2, 2002	3.0	1.425
December 2, 2002	1.5	0.325
December 2, 2002	2.0	0.65
December 2, 2002	2.5	1.05
December 2, 2002	3.0	1.475
December 2, 2002	3.5	1.95
December 2, 2002	24.0	2.45
December 2, 2002	4.5	2.925
December 2, 2002	5.0	3.375
December 2, 2002	6.0	4.40
December 2, 2002	7.0	5.425
March 3, 2003	1.5	0.45
March 3, 2003	2.0	0.775
March 3, 2003	2.5	1.15
March 3, 2003	3.0	1.575
March 3, 2003	3.5	2

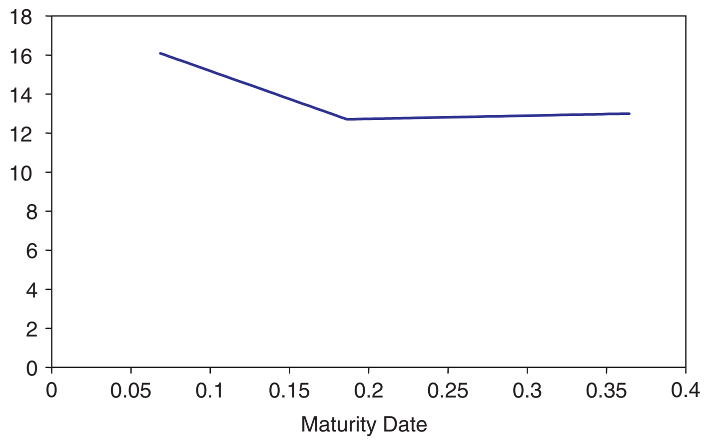

Figure 1: Implied downside transition rate.

The role of parameter $\hat{\eta}$ is critical in the framework and $\hat{\eta}$ can be estimated from market prices of LEPO put options or other traded options with the same underlying as that of the option being priced.

Just like the volatility smile in the Black-Scholes model, from Figure 1 we see that the implied $\hat{\eta}$ from (3.8) of the Korn-Kreer-Lenssen model is also not a constant but dependent on the maturity date.

TABLE II:

Maturity date	Strike price in dime unit	Implied downside intensity $\hat{\eta}$	Theoretical price of put options from BD Model in dime unit	Theoretical price of put options from BD Model in dollar unit
October 2, 2002	15	16.0928	2.25	0.225
October 2, 2002	20		5.6736	0.56736
October 2, 2002	25		10.1089	1.01089
October 2, 2002	30		14.9462	1.49462
December 2, 2002	15	12.7109	3.25	0.325
December 2, 2002	20		6.4985	0.64985
December 2, 2002	25		10.5466	1.05466
December 2, 2002	30		15.0682	1.50682
December 2, 2002	35		19.8342	1.98342
December 2, 2002	40		24.7135	2.47135
December 2, 2002	45		29.6413	2.96413
December 2, 2002	50		34.5884	3.45884
December 2, 2002	60		44.4997	4.44997
December 2, 2002	70		54.4157	5.44157
March 3, 2003	15	13	4.5	0.45
March 3, 2003	20		7.6738	0.76738
March 3, 2003	25		11.4355	1.14355
March 3, 2003	30		15.6154	1.56154
March 3, 2003	35		20.077	2.0077

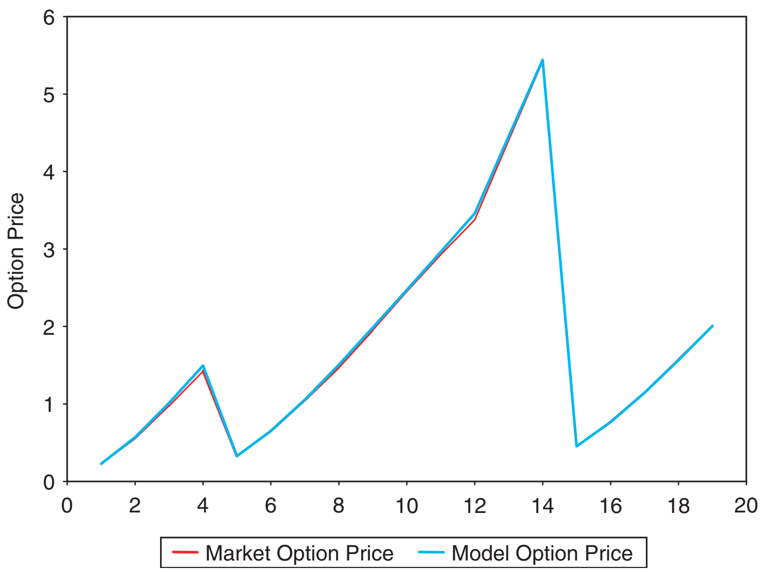

Figure 2: Market option price vs. model option price.

On the maturity date October 2, 2002, using the data in the first row of Table 1, by trial and error in (3.8), we get the implied $\hat{\eta} = 16.0928$. If using Matlab function **fzero** or **lsqnonlin** to estimate $\hat{\eta}$, we get a number very close to 16.0928.

From the estimated $\hat{\eta}$, α and β can be calculated from formula (3.5). From the formula (3.8), by using Matlab programming, we get the theoretical prices 2.25, 5.6736, 10.1089 and 14.9462 for the put options with maturity date October 2, 2002 at various strike prices 15, 20, 25, 30 in dime unit respectively.

Doing similar calculations for other maturity dates, we get Table 2.

Comparing Table 1, Table 2 and see Figure 2 , we find that the market option price and the model option price for the European put option of the company are very close. Moreover, the Korn-Kreer-Lenssen formula (3.8) is relatively simple for calculation.

5. Conclusions

This paper demonstrates implementation and testing of the Korn-Kreer-Lenssen model for European vanilla options by an illustrative example. The model assumes that the underlying stock price follows a linear birth-death process rather than the usual geometric Brownian motion. The option prices derived from the Korn-Kreer-Lenssen model have been demonstrated to be very close to the market option prices of European vanilla options proving the accuracy of this model. The Korn-Kreer-Lenssen formula (3.8) is relatively simple for calculation. In this paper, we also derived two closed form solutions for American digital options in the Korn-Kreer-Lenssen's framework.

It would be interesting to compare the market prices of American digital options with pricing formulas (3.20) and (3.24) derived in this paper. The analysis will be left for future research.

Appendix A

Option Pricing Partial Differential-Difference Equations
When Underlying Stock Returns are Discontinuous

See Merton (1992) from p.318 to p.324 for more details. When underlying stock returns are discontinuous, the Black-Scholes "no arbitrage" technique cannot be directly employed. A different approach to the pricing problem follows along the lines of the original Black-Scholes derivation which assumed that the Capital Asset Pricing Model (CAPM) was a valid description of equilibrium security returns. The stock price jump dynamic is a reflection of important new information that has an instantaneous nonmarginal impact on the stock. If this type of information is usually firm specific, then it may have little impact on stocks in general.

If the source of the jumps is such information, the return of the stock will represent nonsystematic risk. Hence the jump will be uncorrelated with the market. Suppose that this is generally true for stocks.

Consider a portfolio strategy which holds the stock, the option, and the riskless asset with return r per unit time in proportions w_1^*, w_2^* and w_3^* where $\sum_{j=1}^{3} w_j^* = 1$. Suppose P^* be the value of the portfolio. The only source of uncertainty in the return of the portfolio is the jumps of the stock. But by hypothesis, such jumps represent only nonsystematic risk and therefor the "beta" of this portfolio is zero. If the CAPM holds, then the expected return on all zero-beta securities must equal the riskless rate r. Then follow the derivations at p.319 of Merton (1992), we get partial differential-difference equations for option price F:

$$\frac{1}{2}\sigma^2 S^2 F_{SS} + (r - \lambda k)SF_S - F_\tau - rF + \lambda E\{F(SY, \tau) - F(S, \tau)\} = 0 \quad \text{(A.1)}$$

subject to the boundary conditions

$$F(0, \tau) = 0 \quad \text{(A.2)}$$

$$F(S, 0) = \max(0, S - K) \quad \text{(A.3)}$$

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