Please solve the following problems dealing with topics in linear programming. Show all work and answers on your own paper. Answers without justifying work will receive no credit. Partial credit will be given as appropriate, do not leave any problem blank. Your results and supporting work are due at class time on Friday, August 5, 2005.

1. Find the vertices of the feasible set \( x + 2y + 3z = 6, \ x \geq 0, \ y \geq 0, \ z \geq 0 \). Maximize and minimize \( 2x + 7y + 5z \) over this set.

The vertices of the feasible set lie at the points with coordinates \((6, 0, 0), (0, 3, 0), \) and \((0, 0, 2)\). The maximum of \( 2x + 7y + 5z \) is 21 and occurs at the point \((0, 3, 0)\). The minimum of \( 2x + 7y + 5z \) is 10 and occurs at the point \((0, 0, 2)\).

2. For what values of \( a \) does \( x + ay = -1, x \geq 0, y \geq 0 \) give an empty feasible set? For what values of \( a \) is the feasible set unbounded?

If \( a \geq 0 \) the feasible set is empty since no portion of the line \( x + ay = -1 \) lies in the first quadrant. If \( a < 0 \) the feasible set is unbounded.

3. If \( x + y + 3z \) is minimized subject to \( x + y + z = 1, z \geq 0 \), show that two points of the feasible set give the same minimum where \( x + y + 3z = 1 \). Find all the solutions \((x, y, z)\) and explain geometrically how more than one solution is possible.

Every point on the line of the form \( \langle x, 1 - x, 0 \rangle \) for an arbitrary \( x \) will yield a cost of 1. This line of solutions occurs due to the intersection of the planes \( x + y + z = 1 \) and \( x + y + 3z = k \) along a line when \( k = 1 \).
4. Sketch the feasible set for \( x + 2y \geq 6, \ 2x + y \geq 6, \ x \geq 0, \ y \geq 0 \). What points lie at the corners of this set?

The corners of the unbounded feasible set lie at the points with coordinates (0,6), (2,2), and (6,0).

5. On the preceding feasible set, minimize the expression \( x + y \). Draw the line \( x + y \) that first touches the feasible set.

The level sets of the function \( x + y \) are lines with slope \(-1\) and \( y\)-intercept \( k \). The first such line to intersect the feasible region is the one with \( k = 4 \). Thus the minimum of \( x + y \) is 4 and this occurs at the point with coordinates (2, 2).