Please answer the following questions. Unless otherwise stated, all interest rates mentioned below are annual interest rates. Your results and supporting work are due at class time on Friday, July 29, 2005.

1. Find the effective interest rates for:

(a) 3% compounded monthly.

\[
(1 + \frac{0.03}{12})^{12} = 1 + r_e
\]

\[
r_e \approx 0.030416
\]

(b) 18% compounded monthly.

\[
(1 + \frac{0.18}{12})^{12} = 1 + r_e
\]

\[
r_e \approx 0.195618
\]

(c) 18% compounded quarterly.

\[
(1 + \frac{0.18}{4})^{4} = 1 + r_e
\]

\[
r_e \approx 0.192519
\]

2. A major lottery advertises that it pays the winner $10 million. However, this prize money is paid at the rate of $500,000 each year (with the first payment coming immediately) for a total of 20 payments. What is the present value of this prize at 10% interest?

\[
P = 500000 \sum_{i=0}^{19} (1 + 0.10)^{-i}
\]

\[
\approx 4,682,460
\]

3. A student has made a nonrefundable deposit of the first month’s rent ($1000) on a six-month apartment lease. The next day they find a different apartment they like just as much as the first, but its monthly rent is only $900. They plan to be in the apartment for only six months. Assuming an interest rate of 12%, should they switch to the new apartment? What if they plan to stay one year?

Stay for six months:

- in first apartment, present value of all money paid is

\[
P = 1000 \sum_{i=0}^{5} (1 + \frac{0.12}{12})^{-i} \approx 5853.43.
\]

- move to second apartment, present value of all money paid is

\[
P = 1000 + 900 \sum_{i=0}^{5} (1 + \frac{0.12}{12})^{-i} \approx 6268.09.
\]
Thus the student should stay in the first apartment.

Stay for twelve months:
- in first apartment, present value of all money paid is
  \[ P = 1000 \sum_{i=0}^{11} (1 + \frac{0.12}{12})^{-i} \approx 11367.60. \]
- move to second apartment, present value of all money paid is
  \[ P = 1000 + 900 \sum_{i=0}^{11} (1 + \frac{0.12}{12})^{-i} \approx 11230.90. \]

Thus the student should move to the second apartment.

4. You are considering the purchase of a house. It is perfect in every way for you and in excellent condition except for the roof which has only five years of useful life remaining. A new roof will last twenty years and cost $20,000. The rest of the house will last forever. Assuming that costs will remain constant and that the interest rate is 5%, what value would you assign the existing roof?

Imagine the roof loses one twentieth of its value every year. If you borrow $20,000 to install a new roof and make twenty equal yearly payments then the amount of the yearly payment is
\[
20000 = A \sum_{i=1}^{20} (1 + 0.05)^{-i}
\]
\[ A \approx 1604.85 \]

The value of the current roof is the present value of the last five payments of such a borrowing plan. Thus the value of the roof is
\[
V = 1604.85 \sum_{i=16}^{20} (1 + 0.05)^{-i} \approx 3342.18
\]

5. Consider the two enterprises whose income streams are shown in the table below. A negative income means that the owner of the enterprise must pay out rather than receive income.

<table>
<thead>
<tr>
<th>Years</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enterprise 1</td>
<td>-100</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Enterprise 2</td>
<td>-150</td>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
</tr>
</tbody>
</table>

(a) Find the present value of each enterprise at an interest rate of 5%.

\[
P_1 = -100 + 30 \sum_{i=1}^{5} (1 + 0.05)^{-i} \approx 29.8843
\]
\[
P_2 = -150 + 42 \sum_{i=1}^{5} (1 + 0.05)^{-i} \approx 31.838
\]

(b) Find the rate of return of each enterprise.

\[
100 = 30 \sum_{i=1}^{5} (1 + r_1)^{-i} \Rightarrow r_1 \approx 0.152382
\]
\[
150 = 42 \sum_{i=1}^{5} (1 + r_2)^{-i} \Rightarrow r_2 \approx 0.123762
\]
(c) Which enterprise is the better investment opportunity?

The answer depends on which is more important to the investor, the present value of all payments or the rate of return. If the present value is the more important consideration, then Enterprise 2 is more valuable. If the rate of return is the more important consideration, then Enterprise 1 is more valuable.