Eastern North Pacific Gray Whale Census Estimates

An Application of State Space Reconstruction

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Gray Whale

- Latin name *Eschrichtius robustus*, length 40–50 feet (12–15 m), weight 50,000–80,000 lbs (23,000-36,000 kg), lifespan of up to 50 years.
Brief History

- Hunted by aboriginal peoples as well as 19th and 20th century commercial whaling industry
- Believed “commercially extinct” in 1900
- Western Pacific stock nearly extinct (approximately 50 individuals)
Habitat

- Usually found within 2 km of coastline.
- Eastern Pacific females calve in the lagoons of Baja peninsula
Migration

The black dots show the North-south migration path.

The blue lines show the summer and winter homes of the Gray Whale.
Feeding Habits

- Only “bottom feeding” whale
- Prefer to feed with their right sides toward the bottom
- Dredge bottom mud for amphipods and crustaceans
- Have baleen for filtering food from mud
Recovery?

Q: Has the stock of eastern Pacific gray whales recovered sufficiently that regulatory protection is no longer needed?

- Yearly census results continue to oscillate
- Mathematical models exhibit a decrease in population during 1968–1988 while census numbers showed an increase
- Models do not exhibit depletion of stock by 1900

Q: Can the behavior of mathematical models be reconciled with the census data?
Global Model

From the work of de la Mare (1989) and Punt and Butterworth (1991):

\[ P_{t+1} = (P_t - C_t)e^{-M} + (1 - e^{-M})P_{t-t_m+1} \left( 1 + A \left[ 1 - \left( \frac{P_{t-t_m+1}}{P_0} \right)^{z} \right] \right) \]

- \( C_t \)  catch in year \( t \)
- \( M \)  natural mortality rate
- \( t_m \)  age at first parturition
- \( A \)  resilience parameter
- \( z \)  density-dependent exponent
Local Model

Characteristics of local models:

- Local models received much attention from 1970’s through 1990’s
- Found application in predicting time series
- Data driven – no ecological fidelity
- Computationally expensive
- Often better at predicting behavior of chaotic dynamical systems (*e.g.* solutions of the Lorenz equation) than global models
Brief Introduction to Local Modeling

Difference equation model:

\[ P_t = F(P_{t-1}, P_{t-2}, \ldots, P_{t-j}) \]

Sequence of observations over time:

\[ \{P_0, P_1, \ldots, P_N\} \]

Time delay embedding with dimension \( m \), Takens (1981):

\[ x_{m-1} = \langle P_0, P_1, \ldots, P_{m-1} \rangle \]
\[ x_m = \langle P_1, P_2, \ldots, P_m \rangle \]
\[ \vdots \]
\[ x_N = \langle P_{N-m+1}, P_{N-m+2}, \ldots, P_N \rangle \]
Takens’s Theorem

From Casdagli, et al. (1991):

Dynamical system: \( x(t) = f^t(x(0)), \quad x \in \mathbb{R}^n \)

Observable: \( y(t) = g(x(t)), \quad y \in \mathbb{R}^d \)

Delay construction map:

\[ \Phi(x(t)) = \langle g(f^{\tau m_p}(x(t))), \ldots, g(x(t)), \ldots, g(f^{\tau m_f}(x(t))) \rangle \]

If \( m = m_f + m_p + 1 \geq 2n + 1 \) then \( \Phi \) is a smooth, one-to-one coordinate transformation with a smooth inverse.
Local Modeling 2

Prediction:
\[ \hat{P}_{N+1} = G(x_N) \]

Multi-step prediction:
\[
\begin{align*}
\hat{P}_{N+2} &= G(\langle P_{N-m+2}, P_{N-m+3}, \ldots, \hat{P}_{N+1} \rangle) \\
\hat{P}_{N+3} &= G(\langle P_{N-m+3}, P_{N-m+4}, \ldots, \hat{P}_{N+2} \rangle) \\
\vdots
\end{align*}
\]
Fourier Spectrum
Interpolation

Up-sample to fill in the spacing of the sequence of observations.
Filtering

- Remove high frequency noise
- Reduce data storage requirements
- Perform in parallel with interpolation

\[ x_i = L_3 \circ L_2 \circ L_1 (\langle P_{i-w+1}, P_{i-w+2}, \ldots, P_i \rangle) \]

- \( L_1 \) Fourier transform
- \( L_2 \) Low pass filter \( m/2 \) frequencies
- \( L_3 \) Inverse Fourier transform
Nearest Neighbors

Metric:

\[ d^2(x_a, x_b) = \sum_{i=0}^{m-1} \lambda^i (x_{a,m-i} - x_{b,m-i})^2 \]

with \( 0 < \lambda \leq 1 \).
Prediction Algorithm

1. Create filtered embedding of sequence of observations
2. Find $k \geq 1$ nearest neighbors of $x_N$
3. For neighbors $\{x_{n_1}, \ldots, x_{n_k}\}$ find $\{P_{n_1+1}, \ldots, P_{n_k+1}\}$
4. Approximate the map $x_\alpha \xrightarrow{G} P_{\alpha+1}$
5. Evaluate $G(x_N) = \hat{P}_{N+1}$
Types of Maps: Averaging

Direct:

\[
\hat{P}_{N+1} = \frac{\sum_{i=1}^{k} w_i P_{n_i+1}}{\sum_{i=1}^{k} w_i}
\]

Integrated:

\[
\hat{P}_{N+1} = P_N + \frac{\sum_{i=1}^{k} w_i (P_{n_i+1} - P_{n_i})}{\sum_{i=1}^{k} w_i}
\]

Weights depend on the distance between the neighboring vector and the query vector.

\[
\hat{w_i} = \left[ 1 - \left( \frac{d(x_N, x_{n_i})}{d(x_N, x_{n_k})} \right)^2 \right]^2
\]
Another Type: Linear

From Sauer (1994):

1. Let \( c \) be the center of mass of \( \{x_{n_1}, \ldots, x_{n_k}\} \)

2. For some \( l \leq m \) find the \( l \)-dimensional subspace of \( \mathbb{R}^m \) containing \( c \) closest to the span of \( \{x_{n_1}, \ldots, x_{n_k}\} \)

\[
A = \begin{bmatrix}
x_{n_1} - c \\
\vdots \\
x_{n_k} - c
\end{bmatrix} = U^tDV
\]

3. Project \( \{x_{n_1} - c, \ldots, x_{n_k} - c\} \) onto \( \mathbb{R}^l \)

4. Find the affine map \( G : \mathbb{R}^l \to \mathbb{R} \) which best fits the data \( \{(\Pi(x_{n_1} - c), P_{n_1+1}), \ldots, (\Pi(x_{n_k} - c), P_{n_k+1})\} \)

\[
\hat{P}_{N+1} \approx G(\Pi(x_N - c))
\]
Constant Model, Direct Averaging

Parameters: Euclidean metric, 2 nearest neighbors, embedding dimension 32
Constant Model, Integrated Averaging

Parameters: Euclidean metric, 2 nearest neighbors, embedding dimension 32
Parameters: Euclidean metric, 4 nearest neighbors, embedding dimension 32, model dimension 2
Overlay of Model Results

Model: integrated, averaged, linear
Sensitivity to Nearest Neighbors

Number of nearest neighbors: 2, 3, 4, 5
Comments

- Linear model requires more neighbors than the constant models.
- Linear model requires more data than the constant models.
- Constant models exhibit plausible oscillatory behavior.
- Population levels predicted by the local models exhibit flat average behavior
- Need much more data for model validation.
- Has the gray whale population been above its carrying capacity?
References


