Addition and Subtraction with Polynomials

MATH 101 College Algebra

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Objectives

In this lesson we will learn to:

- identify polynomial expressions,
- classify certain polynomials as monomials, binomials, or trinomials,
- add and subtract polynomials,
- evaluate polynomials for given values of the variables.
Monomials

**Definition**

A **monomial in** $x$ is an expression of the form

$$kx^n$$

where $n$ is a positive whole number and $k$ is any real number. Exponent $n$ is called the **degree** of the monomial and $k$ is called the **coefficient**.
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Comments:
- A monomial may have more than one variable. In this case the degree is the sum of the exponents of the variables.
- A nonzero constant monomial has degree 0 (since $k = k \cdot x^0$).
- The constant 0 has no degree.
A **polynomial** is a monomial or the algebraic sum or difference of monomials.

The **degree of a polynomial** is the largest of the degrees of its terms after like terms have been combined.

The coefficient of the term with the largest degree is called the **leading coefficient**.

Polynomials with one, two, or three terms are called **monomials**, **binomials**, and **trinomials** respectively.

Polynomials of degree 0 or 1 are called **linear**, of degree 2 are called **quadratic**, and of degree 3 are called **cubic**.
We will write polynomials left to right in descending order of degree, for example

\[3x + 5x^2 - 10 + \frac{1}{3}x^3 = \frac{1}{3}x^3 + 5x^2 + 3x - 10.\]
Add polynomials by combining like terms.
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Example

\[(3x^2 - 2x + 5) + (2x^2 - x + 3)\]
Add polynomials by combining like terms.

Example

\[
(3x^2 - 2x + 5) + (2x^2 - x + 3) = 3x^2 + 2x^2 - 2x - x + 5 + 3 = 5x^2 - 3x + 8
\]
To find the difference of two polynomials add the opposite of each term being subtracted.
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Example

$$(5x^3 - 9x^2 - 10x + 12) - (3x^3 + 6x^2 - 7)$$
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\[(5x^3 - 9x^2 - 10x + 12) - (3x^3 + 6x^2 - 7)\]
\[= 5x^3 - 3x^3 - 9x^2 - 6x^2 - 10x + 12 + 7\]
\[= 2x^3 - 15x^2 - 10x + 19\]
We will sometimes use function notation when dealing with polynomials.

\[ P(x) = x^3 - 8x^2 - 5x + 10 \]
\[ P(x, y) = 2x^2y - xy + 3x - 4y + 1 \]

These functions are read as “\( P \) of \( x \)” and as “\( P \) of \( x \) and \( y \)”.
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To evaluate the polynomial at a number, substitute the number for the variable of the polynomial.