Please answer the following questions. Your answers will be evaluated on their correctness, completeness, and use of statistical concepts we have covered. Please show all work and write out your work neatly. Answers without supporting work will receive no credit. The point values of the problems are listed in parentheses.

1. (3 points each) Benjamin owns a small business. Besides himself, he employs nine other people. The salaries of the people employed by Benjamin are shown in the table below (units of thousands of dollars).

\[
20 \quad 35 \quad 45 \quad 50 \quad 50 \quad 50 \quad 55 \quad 55 \quad 60 \quad 80
\]

(a) Find the mean salary.

\[
\mu = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{500}{10} = 50.0
\]

(b) Find the median salary.

\[
M = \frac{50 + 50}{2} = 50.0
\]

(c) Find the mode salary, if there is a mode.

mode = 50

(d) Find the midrange salary.

\[
\text{midrange} = \frac{20 + 80}{2} = 50.0
\]
2. (3 points each) In a random sample of 250 toner cartridges, the mean number of pages a toner cartridge can print is 4302 and the standard deviation is 340.

(a) Consider the histogram of the data shown below. Describe the shape of the distribution.

The distribution is symmetric and approximately bell-shaped.

(b) Assuming the Empirical Rule can be used, 95% of toner cartridges will be print between what two number of pages? Specify the range (low to high) for the 95%.

The middle 95% of the pages printed will be within 2 standard deviations of the mean.

\[(\bar{x} - 2s, \bar{x} + 2s) = (4302 - 2(340), 4302 + 2(340)) = (3622, 4982)\]

(c) Assuming the Empirical Rule can be used, what percentage of toner cartridges will print less than 3622 pages?

The Empirical Rule states that 5% of the pages printed will be outside of the interval found above. Due to the symmetry of the distribution of page printed, 2.5% of the toner cartridges will print less than 3622 pages.

(d) Using Chebyshev’s Inequality determine the minimum percentage of toner cartridges that print between 3282 and 5322 pages.

The page counts 3282 and 5322 are 3 standard deviations away from the mean.

\[\left(1 - \frac{1}{3^2}\right) \cdot 100\% = 88.9\%\]
3. (4 points each) Suppose a compact disk (CD) you just purchased has 17 songs. After listening to the CD, you decide you like 6 of the songs. The shuffle feature of your CD player will play each of the 17 songs once in a random order.

   (a) Find the probability that among the first 5 songs played that you liked 2 of the songs.

   \[
   p = \frac{\binom{6}{2} \binom{11}{3}}{\binom{17}{5}}
   \]

   \[
   = \frac{(15)(165)}{6188}
   \]

   \[
   = 0.4000
   \]

   (b) Find the probability that among the first 5 songs played that you liked all 5 of the songs.

   \[
   p = \frac{6 \binom{5}{5}}{\binom{17}{5}}
   \]

   \[
   = \frac{6}{6188}
   \]

   \[
   = 0.0009696
   \]

4. (3 points) If \( P(A) = 0.450 \), \( P(A \text{ or } B) = 0.400 \), and \( P(A \text{ and } B) = 0.200 \), find \( P(B) \).

   Using the general form of the Addition Rule we have

   \[
   P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)
   \]

   \[
   0.400 = 0.450 + P(B) - 0.200
   \]

   \[
   P(B) = 0.400 + 0.200 - 0.450
   \]

   \[
   P(B) = 0.150.
   \]
5. (3 points) Suppose 60 cars start a race. In how many ways can the fastest 3 cars finish the race?

\[ 60P_3 = \frac{60!}{(60 - 3)!} = \frac{60!}{57!} = 205320 \]

6. (3 points each) The data in the following table represent a sample of ATM fees charged by banks in two different cities.

<table>
<thead>
<tr>
<th>City</th>
<th>1.25</th>
<th>1.00</th>
<th>1.50</th>
<th>1.25</th>
<th>1.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>City A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>City B</td>
<td>2.25</td>
<td>1.50</td>
<td>1.75</td>
<td>0.00</td>
<td>2.00</td>
</tr>
</tbody>
</table>

For City A, \( \sum x_i = 6.25 \) and \( \sum x_i^2 = 7.9375 \). For City B, \( \sum x_i = 7.50 \) and \( \sum x_i^2 = 14.375 \).

(a) What is the range in ATM fees for city A?

\[ R = 1.50 - 1.00 = 0.50 \]

(b) What is the range in ATM fees for city B?

\[ R = 2.25 - 0.00 = 2.25 \]

(c) What is the standard deviation in ATM fees for city A?

\[
\begin{align*}
    s^2 &= \frac{\sum x_i^2 - \left( \frac{\sum x_i}{n} \right)^2}{n - 1} \\
    &= \frac{7.9375 - \left( \frac{6.25}{5} \right)^2}{5 - 1} \\
    &= 0.03215 \\
    s &= \sqrt{0.03215} \approx 0.177
\end{align*}
\]
(d) What is the standard deviation in ATM fees for city B?

\[ s^2 = \frac{\sum x_i^2 - \left( \frac{\sum x_i}{n} \right)^2}{n - 1} = \frac{14.375 - (7.50)^2}{5 - 1} \]

\[ s^2 = 0.78125 \]

\[ s = \sqrt{0.78125} \approx 0.884 \]

(e) Based on range, which city has the greater dispersion in ATM fees?

City B has the greater range.

(f) Based on standard deviation, which city has the greater dispersion in ATM fees?

City B has the greater standard deviation.

7. (3 points each) The data in the following table represent the number of driving fatalities for a certain state by age for male and female drivers.

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>under 16</td>
<td>198</td>
<td>140</td>
</tr>
<tr>
<td>16–20</td>
<td>5045</td>
<td>2418</td>
</tr>
<tr>
<td>21–34</td>
<td>13950</td>
<td>4056</td>
</tr>
<tr>
<td>35–54</td>
<td>10152</td>
<td>5282</td>
</tr>
<tr>
<td>55–69</td>
<td>4649</td>
<td>1868</td>
</tr>
<tr>
<td>70 and over</td>
<td>3233</td>
<td>1447</td>
</tr>
</tbody>
</table>

(a) What is the probability that a randomly selected driver fatality who was male, was 35 to 54 years old?

\[ P(35 - 54 \mid \text{male}) = \frac{10152}{37227} = 0.273 \]
(b) What is the probability that a randomly selected driver fatality who was 35 to 54 years old, was male?

\[ P(\text{male} \mid 35-54) = \frac{10152}{10152 + 5282} = 0.658 \]

(c) Is a victim of a fatal accident aged 35 to 54 more likely to be male or female? Justify your answer.

The probability of being a male victim is higher.
8. (4 points each) The numbers in the table below represent the inches of rainfall for a city over the past 20 years.

<table>
<thead>
<tr>
<th></th>
<th>0.22</th>
<th>0.67</th>
<th>1.05</th>
<th>1.36</th>
<th>1.73</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.85</td>
<td>1.98</td>
<td>2.37</td>
<td>2.61</td>
<td>2.74</td>
</tr>
<tr>
<td></td>
<td>2.88</td>
<td>2.99</td>
<td>3.13</td>
<td>3.38</td>
<td>3.71</td>
</tr>
<tr>
<td></td>
<td>4.16</td>
<td>4.54</td>
<td>4.87</td>
<td>5.33</td>
<td>5.59</td>
</tr>
</tbody>
</table>

(a) Find the five-number summary for the table.

<table>
<thead>
<tr>
<th>minimum</th>
<th>$Q_1$</th>
<th>$M$</th>
<th>$Q_3$</th>
<th>maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.22</td>
<td>1.79</td>
<td>2.81</td>
<td>3.935</td>
<td>5.59</td>
</tr>
</tbody>
</table>

(b) What is the interquartile range?

$$IQR = 3.395 - 1.79 = 2.145$$

(c) Find the lower and upper fences for the data.

lower fence = $Q_1 - 1.5(IQR) = 1.79 - 1.5(2.145) = -1.4275$

upper fence = $Q_3 + 1.5(IQR) = 3.935 + 1.5(2.145) = 7.1525$

(d) List any outliers in the data, or write “NONE” if there are no outliers.

NONE
9. (3 points) Suppose that $A$ and $B$ are two events and that $P(A \text{ and } B) = 0.2$ and $P(A) = 0.5$. What is $P(B \mid A)$?

Using the Multiplication Rule for dependent events we have

$$P(A \text{ and } B) = P(A)P(B \mid A)$$

$$0.2 = 0.5P(B \mid A)$$

$$P(B \mid A) = \frac{0.2}{0.5} = 0.400$$

10. (4 points each) Containers of milk can be classified as either raw or pasteurized. 81% of milk sold in containers in stores is classified as pasteurized.

(a) What is the probability that two randomly selected containers of milk are classified as pasteurized?

Since the two containers are randomly selected, the selections are independent events.

$$p = (0.81)(0.81) = 0.6561$$

(b) What is the probability that at least one of six randomly selected containers of milk is classified as raw?

$$P(\text{at least 1}) = 1 - P(\text{none})$$

$$= 1 - (0.81)^6$$

$$= 0.7176$$
11. (2 points each) The following boxplot was created from a data set of weights of quarter dollar coins (measured in grams).

(a) Describe the shape of the distribution of the weights of the coins.
   The distribution is skewed right.

(b) Estimate to the nearest 0.1 gram, the median weight of the coins.

\[ M = 5.6 \]

(c) Estimate to the nearest 0.1 gram, the first quartile weight of the coins.

\[ Q_1 = 5.58 \]

(d) Would a coin weighing 5.90 grams be considered an outlier? Justify your answer.
   Yes, because 5.90 lies to the right of the upper fence.