Please answer the following questions. Your answers will be evaluated on their correctness, completeness, and use of mathematical concepts we have covered. Please show all work and write out your work neatly. Answers without supporting work will receive no credit. The point values of the problems are listed in parentheses.

1. (9 points) Use a known Taylor series to evaluate the following definite integral.

\[ \int_{-1}^{1} e^{-x^2} \, dx \]

2. (9 points) Use a known Taylor series to find the Taylor series with \( c = 0 \) for the function \( f(x) = x \sin 2x \).
3. (10 points) Find the Maclaurin series for sinh \( x \).
4. (9 points) Determine the interval of convergence and the function to which the following series converges.

\[ \sum_{k=0}^{\infty} (-1)^k \left( \frac{x}{2} \right)^k \]

5. (9 points) Find a power series representation for the function \( f(x) = \frac{2}{4+x} \). State the radius of convergence of the power series.
6. (9 points) Determine the interval of convergence of the following power series.

\[ \sum_{k=2}^{\infty} k^2 (x - 3)^k \]

7. (9 points) Find a power series in \( x \) representation of \( f(x) = \frac{2x}{(1 - x^2)^2} \).
8. (9 points each) Determine whether the following infinite series are absolutely convergent, conditionally convergent, or divergent. You must justify your answers by naming the test(s) for convergence or divergence you use.

(a) \[ \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{2k + 1} \]

(b) \[ \sum_{k=1}^{\infty} \frac{k^2}{e^k} \]
9. (9 points) Determine the minimum number of terms necessary to estimate the sum of the following series to within $10^{-4}$. Then using the partial sum with the minimum number of terms, estimate the sum of the series.

\[
\sum_{k=1}^{\infty} \frac{(-1)^k}{k\sqrt{k}}
\]

\[
\sum_{k=1}^{\infty} (-1)^k \frac{3}{k^3}
\]