Please answer the following questions. Your answers will be evaluated on their correctness, completeness, and use of mathematical concepts we have covered. Please show all work and write out your work neatly. Answers without supporting work will receive no credit. The point values of the problems are listed in parentheses.

1. (10 points) Find the exact sum of the series

\[ \sum_{k=1}^{\infty} (-1)^k \frac{4}{7^k}. \]

2. (10 points each) Determine whether the following infinite series converge absolutely, converge conditionally, or diverge. You must justify your answer using a valid test (or tests) for convergence or divergence.

(a) \[ \sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2} \]
(b) \[ \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{\sqrt{k^2+1}}} \]

(c) \[ \sum_{k=1}^{\infty} \frac{(-1)^{k}(k^2 + 1)}{2k^2 + k - 1} \]
(d) \[ \sum_{k=0}^{\infty} \frac{(-3)^k}{k!} \]

3. (10 points) Find the exact sum of the series

\[ \sum_{k=3}^{\infty} \frac{8}{(4k-3)(4k+1)}. \]
4. (10 points each) Determine whether the following sequences converge or diverge. If they converge, find their limits. In each exercise \( n \geq 1 \).

(a) \( a_n = \left( \frac{n-1}{n} \right)^n \)

(b) \( b_n = \frac{\ln(2n^3 + 1)}{n} \)

(c) \( c_n = \frac{1 - (-1)^n}{\sqrt{n}} \)
5. (10 points) Find the radius and interval of convergence of the power series

\[ \sum_{k=0}^{\infty} \frac{(x - \sqrt{2})^{k+1}}{2^k}. \]