1. The region bounded by \( y = \sqrt{\frac{x}{x^2 + 2}} \), the \( x \)-axis, and \( x = 1 \) is revolved around the \( x \)-axis. Find the volume of the solid of revolution.

(a) \( \frac{\pi}{2} \ln 3 \)
(b) \( \frac{\pi}{3} \ln 2 \)
(c) \( \frac{3\pi}{2} \)
(d) \( \frac{2\pi}{3} \)
(e) none of the above.

\[
V = \pi \int_{0}^{1} \left( \sqrt{\frac{x}{x^2 + 2}} \right)^2 dx
= \pi \int_{0}^{1} \frac{x}{x^2 + 2} dx
= \frac{\pi}{2} \ln |x^2 + 2| \bigg|_{0}^{1}
= \frac{\pi}{2} (\ln 3 - \ln 2)
= \frac{\pi}{2} \ln \frac{3}{2}
\]
2. Find the exact value of the arc length of the graph of \( y = \frac{1}{4}x^2 - \frac{1}{2}\ln x \) for \( 1 \leq x \leq 2 \).

(a) \( \frac{3}{4} - \frac{1}{2}\ln 2 \)

(b) \( \frac{3}{4} + \frac{1}{2}\ln 2 \)

(c) \( 1 - \frac{1}{2}\ln 2 \)

(d) \( 1 + \frac{1}{2}\ln 2 \)

(e) none of the above.

\[
\begin{align*}
\mathcal{L} & = \int_1^2 \sqrt{1 + \left( \frac{1}{2}x - \frac{1}{2x} \right)^2} \, dx \\
& = \int_1^2 \sqrt{1 + \frac{1}{4}x^2 - \frac{1}{2} + \frac{1}{4x^2}} \, dx \\
& = \int_1^2 \sqrt{\frac{1}{4}x^2 + \frac{1}{2} + \frac{1}{4x^2}} \, dx \\
& = \int_1^2 \sqrt{\left( \frac{1}{2}x + \frac{1}{2x} \right)^2} \, dx \\
& = \int_1^2 \left( \frac{1}{2}x + \frac{1}{2x} \right) \, dx \\
& = \left. \frac{x^2}{4} + \frac{1}{2}\ln x \right|_1^2 \\
& = \left( 1 + \frac{1}{2}\ln 2 \right) - \left( \frac{1}{4} + \ln 1 \right) \\
& = \frac{3}{4} + \frac{1}{2}\ln 2
\end{align*}
\]
3. A force of 5 pounds stretches a spring 4 inches. Find the work done in stretching this spring 6 inches beyond its natural length.

(a) 10 inch-pounds
(b) 25/2 inch-pounds
(c) 45/2 inch-pounds
(d) 30 inch-pounds
(e) none of the above.

According to Hooke’s Law \( F = kx \) which implies

\[
5 = k(4) \implies k = \frac{5}{4},
\]

Thus the work done is

\[
W = \int_{0}^{6} \frac{5}{4}x\,dx
= \frac{5}{8}x^2\bigg|_{0}^{6}
= \frac{45}{2} \text{ inch-pounds}.
\]
4. An underwater viewing window is installed at an aquarium. The window is circular with radius 5 feet. The center of the window is 40 feet below the surface of the water. Find the hydrostatic force on the window.

(a) 98,018 pounds
(b) 126,562 pounds
(c) 196,035 pounds
(d) 3,141,590 pounds
(e) none of the above.

\[
F = 62.4 \int_{-5}^{5} (40 - y)^2 \sqrt{25 - y^2} \, dy \\
= 124.8 \int_{-5}^{5} (40 - y) \sqrt{25 - y^2} \, dy \\
\approx 196,035 \text{ pounds}.
\]